MATH 30-1

CHAPTER 10 FUNCTION OPERATIONS

$$f(x) + g(x) = (f + g)(x)$$

$$f(x) - g(x) = (f - g)(x)$$

$$f(x) \cdot g(x) = (fg)(x)$$

$$\frac{f(x)}{g(x)} = \left(\frac{f}{g}\right)(x)$$

$$f(g(x)) = (f \circ g)(x)$$

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Unit: **Function Operations**

Topic: **Sums and Differences of Functions**

Objectives:

- Sketching the graph of a function that is the sum or difference of two functions
- Determining the domain and the range of a function that is the sum or difference of two functions.
- Writing the equation of a function that is the sum or difference of two functions

Example

Consider the function $f(x) = x+1$ and $g(x) = 2x-5$. Use the table of values to comare the output values for $y = f(x)$, and $g(x)$ given input values of -2, -1, 0, 1, and 2.

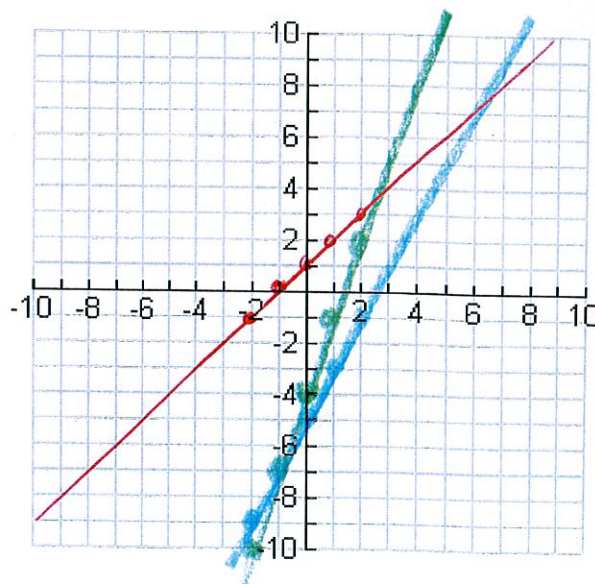| x	$f(x) = x+1$	$g(x) = 2x-5$	$h(x) = 3x-4$
-2	~1	-9	-10
-1	0	-7	-7
0	1	-5	-4
1	2	-3	-1
2	3	-1	2

Sketch the graphs of $y = f(x)$, $y = g(x)$, and $y = h(x)$ on the grid provided.

How could you use the values in the columns $f(x)$ and $g(x)$ to determine the values in the column for $h(x)$? $f(x) + g(x) = h(x)$

How are the y-coordinates on the graph of $h(x)$ related to those on the graphs of $f(x)$ and $g(x)$?

They are the sum of $f(x)$ and $g(x)$.

Fill in the table.

	$f(x)$	$g(x)$	$h(x) = f(x) + g(x)$
Domain	$x \in \mathbb{R}$	$x \in \mathbb{R}$	$x \in \mathbb{R}$

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Example

Consider the function $f(x)=x^2$ and $g(x)=4x-4$. Use the table of values to comare the output values for $y=f(x)$, and $g(x)$ given input values of -2, -1, 0, 1, and 2.

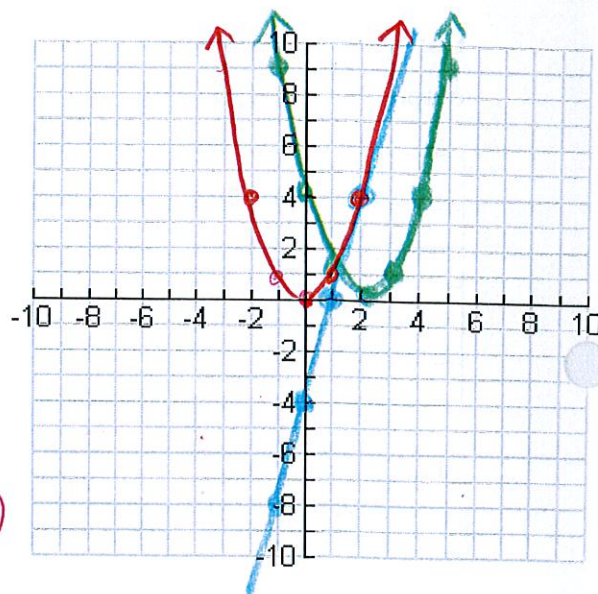| x	$f(x)=x^2$	$g(x)=4x-4$	$h(x)=x^2-4x+4$
-2	4	-12	16
-1	1	-8	9
0	0	-4	4
1	1	0	1
2	4	4	0

Sketch the graphs of $y=f(x)$, $y=g(x)$, and $y=h(x)$ on the grid provided.

For h(x), plug a few more X values in.

How could you use the values in the columns $f(x)$ and $g(x)$ to determine the values in the column for $h(x)$?

$$f(x)-g(x)=h(x)$$

How are the y-coordinates on the graph of $h(x)$ related to those on the graphs of $f(x)$ and $g(x)$?

They are the difference between f(x) and g(x)

Fill in the table.

	$f(x)$	$g(x)$	$h(x)=f(x)+g(x)$
Domain	$x \in \mathbb{R}$	$x \in \mathbb{R}$	$x \in \mathbb{R}$
Range	$y \geq 0$	$y \in \mathbb{R}$	$y \geq 0$

Determine the values of $f(x), g(x),$ and $h(x)$ when x = 3.

$f(x) = 9$

$g(x) = 8$

$h(x) = 1$

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Example

Consider the function $f(x) = |x|$ and $g(x) = x - 2$.

a) Determine the equation of the function $h(x) = f(x) - g(x)$.

$$h(x) = |x| - (x - 2)$$
$$h(x) = |x| - x + 2$$

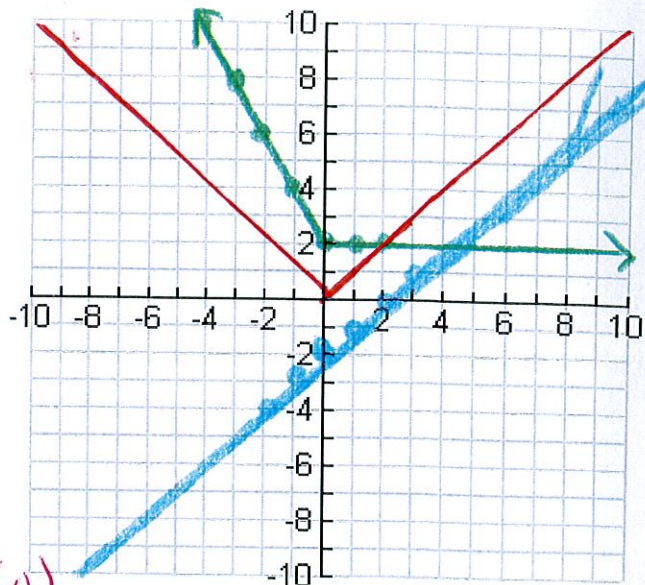b) Sketch the graphs of $f(x)$, $g(x)$, and $h(x)$ on the grid provided.

c) State the domain and range of $h(x)$.

D: $x \in \mathbb{R}$

R: $y \geq 2$

d) Is $h(x) = f(x) - g(x)$ equal to $h(x) = g(x) - f(x)$? If not, what are the similarities and the differences?

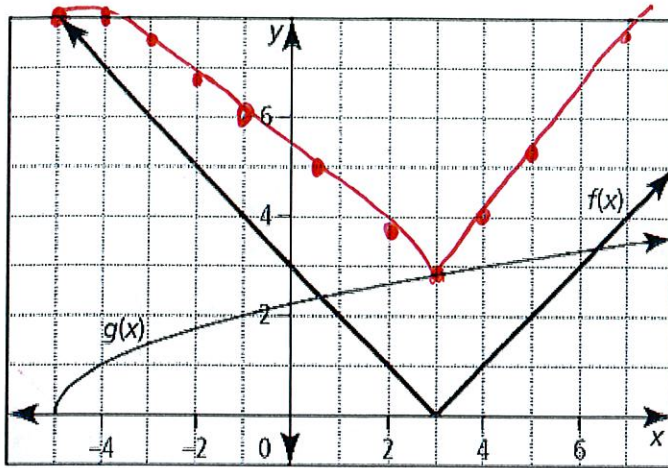| x	$h(x)$
-2	6
-1	4
0	2
1	2
2	2

Between $(f(x) - g(x))$ and $(g(x) - f(x))$, these are vertical reflections of one another.

Similarities: same general shape
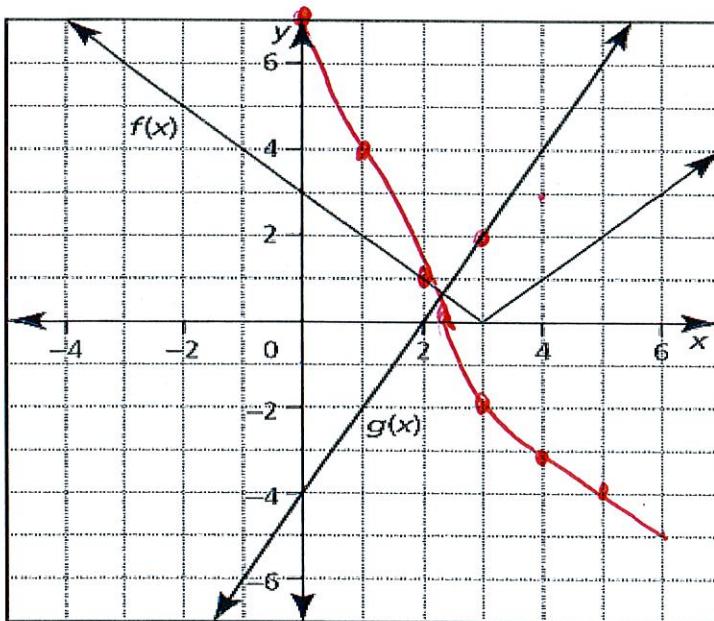
Differences: flipped upside down

Example

Sketch the graph of $h(x) = (f + g)(x)$ given the graphs of $f(x)$ and $g(x)$

Just add the y-components.

Example

Sketch the graph of $h(x) = (f - g)(x)$ given the graphs of $f(x)$ and $g(x)$

Subtract the y-component of $f(x) - g(x)$

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Summary

	Sum of Functions	Difference of Functions
Notation	$h(x) = f(x) + g(x)$ or $h(x) = (f+g)(x)$	$h(x) = f(x) - g(x)$ or $h(x) = (f-g)(x)$
Example	$f(x) = x^2$, $g(x) = -x+5$ $h(x) = f(x) + g(x)$ $h(x) = x^2 + (-x+5)$ $h(x) = x^2 - x + 5$	$f(x) = x^2$, $g(x) = -x+5$ $h(x) = f(x) - g(x)$ $h(x) = x^2 - (-x+5)$ $h(x) = x^2 + x - 5$
Domain	The "overlap" domain common to both graphs.	
Range	Difficult to predict. You'll need to graph it!	

Textbook: page 483 questions 1 to 7, 9, 11, 12

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Unit: Function Operations

Topic: Product and Quotients of Functions

Objectives:

- Sketching the graph of a function that is the product or quotient of two functions
- Determining the domain and the range of a function that is the product or quotient of two functions.
- Writing the equation of a function that is the product or quotient of two functions

Example

Consider the function $f(x) = x$ and $g(x) = -2x + 2$. Use the table of values to comare the output values for $y = f(x)$, and $g(x)$ given input values of -2, -1, 0, 1, and 2.

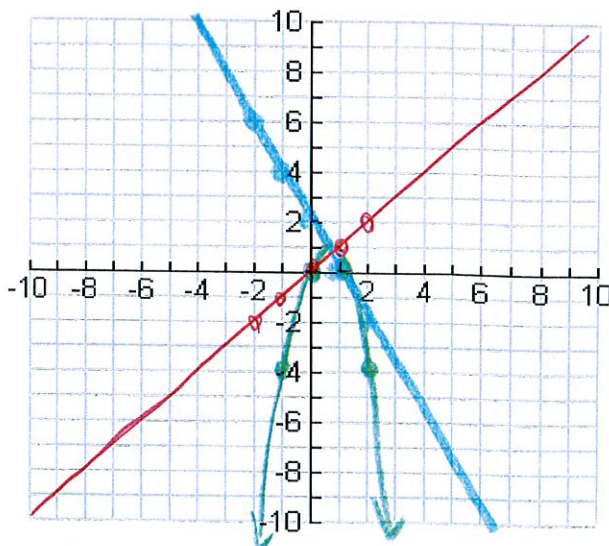| x	$f(x) = x$	$g(x) = -2x + 2$	$h(x) = -2x^2 + 2x$
-2	-2	6	-12
-1	-1	4	-4
0	0	2	0
1	1	0	0
2	2	-2	-4

Sketch the graphs of $y = f(x)$, $y = g(x)$, and $y = h(x)$ on the grid provided.

How could you use the values in the columns $f(x)$ and $g(x)$ to determine the values in the column for $h(x)$?

Multiply them

How are the y-coordinates on the graph of $h(x)$ related to those on the graphs of $f(x)$ and $g(x)$?

The product

Fill in the table.

	$f(x)$	$g(x)$	$h(x) = f(x) \bullet g(x)$
Domain	$x \in \mathbb{R}$	$x \in \mathbb{R}$	$x \in \mathbb{R}$

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Example

Consider the function $f(x) = x^2 + x - 6$ and $g(x) = 2x + 6$. Use the table of values to comare the output values for $y = f(x)$, and $g(x)$ given input values of -2, -1, 0, 1, and 2.

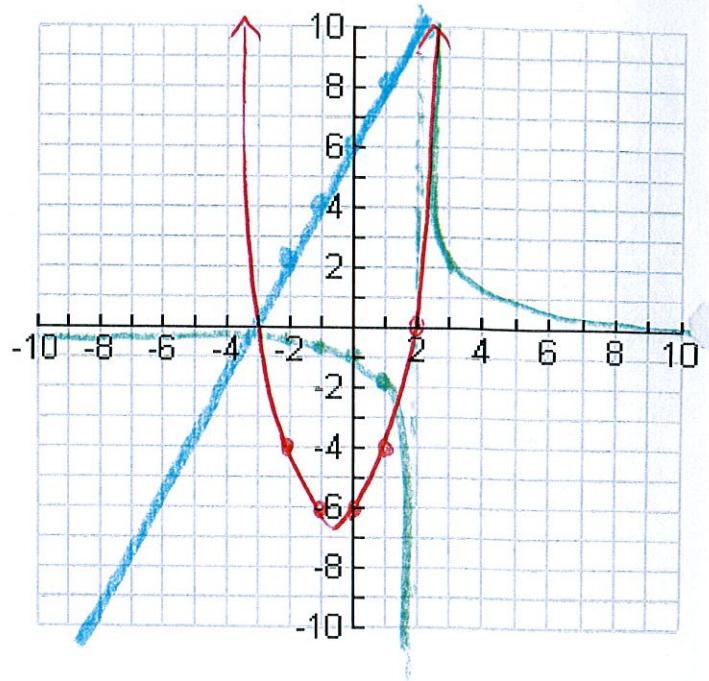| x	$f(x) = x^2 + x - 6$	$g(x) = 2x + 6$	$h(x) = \dfrac{2}{x-2}$
-2	-4	2	-0.5
-1	-6	4	-0.666
0	-6	6	-1
1	-4	8	-2
2	0	10	N/A

Sketch the graphs of $y = f(x)$, $y = g(x)$, and $y = h(x)$ on the grid provided.

How could you use the values in the columns $f(x)$ and $g(x)$ to determine the values in the column for $h(x)$?

$g(x) \div f(x)$

How are the y-coordinates on the graph of $h(x)$ related to those on the graphs of $f(x)$ and $g(x)$?

Quotient of $\dfrac{g(x)}{f(x)}$

Determine the values of $f(x), g(x),$ and $h(x)$ when x = 4.

$f(x) = 14$ $g(x) = 14$ $h(x) = 1$

Fill in the table.

	$f(x)$	$g(x)$	$h(x) = \dfrac{f(x)}{g(x)}$
Domain	X∈ℝ	X∈ℝ	X≠2, X≠-3
Range	y≥-6.25	y∈ℝ	y≠0

Need to find vertex

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Example

Consider the function $f(x) = x + 2$ and $g(x) = x^2 + 9x + 14$.

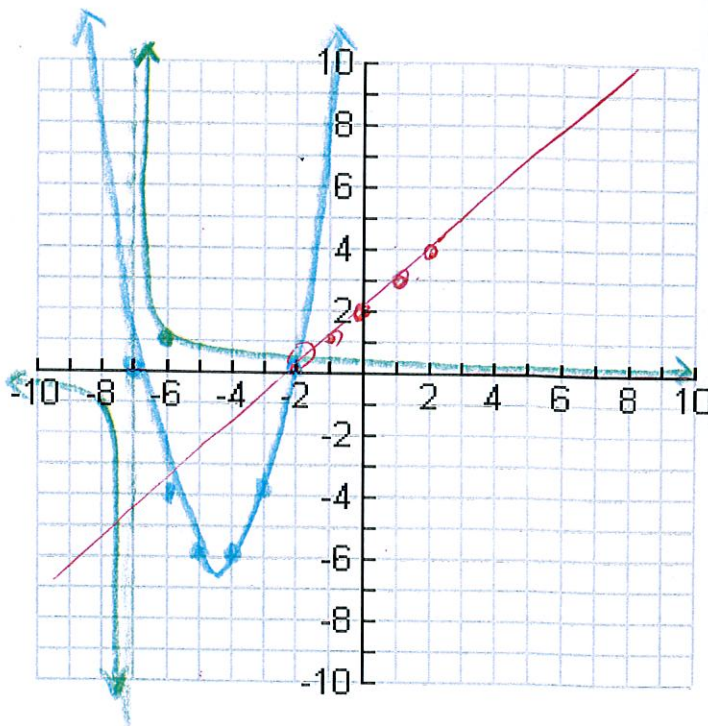a) Determine the equation of the function $h(x) = \left(\dfrac{f}{g}\right)(x)$.

$$h(x) = \frac{x+2}{x^2+9x+14} \rightarrow \frac{x+2}{(x+7)(x+2)} \rightarrow h(x) = \frac{1}{x+7}$$

b) Sketch the graphs of $f(x)$, $g(x)$, and $h(x)$ on the grid provided.

c) State the domain and range of $h(x)$.

D: $x \neq -2, \; x \neq -7$

R: $y \neq 0$

Summary

To combine two functions, $f(x)$ and $g(x)$, multiply or divide as follows:

	Product of Functions	*Quotient of Functions*
Notation	$(f \cdot g)(x)$	$\left(\frac{f}{g}\right)(x)$
Domain	Overlap of domains.	Overlap of domains, $g(x) \neq 0$
Range	Determine with graph	$\longrightarrow$

Textbook: page 496 questions 1 to 8, 12, 14

Math 30-1

Unit: **Function Operations**

Topic: **Composite Function**

Objectives:

- Determining values of a composite function
- Writing the equation of a composite function and explaining any restrictions
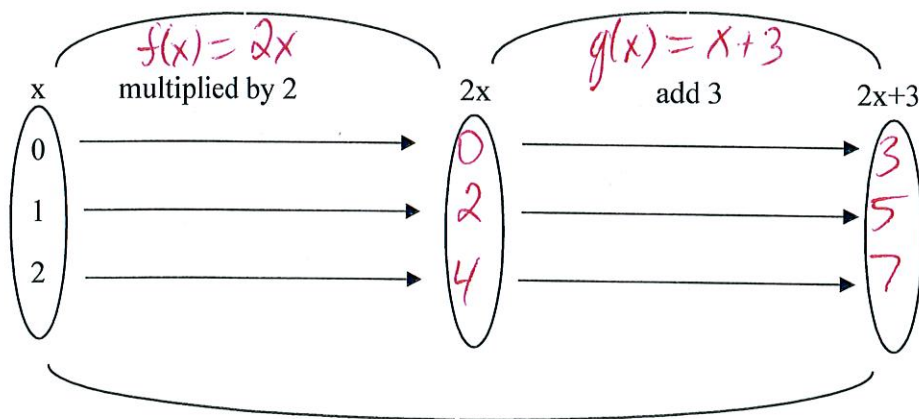- Sketching the graph of a composite function

Consider the function given by $h(x) = 2x + 3$. This function can be thought of as being composed of two functions:

the "multiply by 2" function denoted by f and the "add 3" function denoted by g.

Given that $h(x) = f(x) + g(x)$, determine an equation for $f(x)$ and $g(x)$

$$f(x) = 2x \qquad g(x) = 3. \quad \text{Let's look at it from a different perspective now.}$$

The composite function $h(x)$, says first multiply by 2 and then add 3. Complete the arrow diagram below.

$$f(x) = 2x \qquad\qquad g(x) = x + 3$$

| x | multiplied by 2 | 2x | add 3 | 2x+3 |

x		2x		2x+3
0	→	0	→	3
1	→	2	→	5
2	→	4	→	7

Composite function – a composite function is a "function inside a function". The inner function replaces x in the outer function.

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Denoted by $g(f(x))$ or $(g \circ f)(x)$

at g in f

Put f in g.

Example

If $f(x) = 2x - 2$ and $g(x) = x^2 + 3x$ determine each of the following.

a) $f(g(x))$

b) $g(f(x))$

$2(x^2 + 3x) - 2$

$= \boxed{2x^2 + 6x - 2}$

$(2x-2)^2 + 3(2x-2)$
$= (2x-2)(2x-2) + 6x - 6$
$= 4x^2 - 8x + 4 + 6x - 6$
$= \boxed{4x^2 - 2x - 2}$

Example

If $f(x) = 4 - x$ and $g(x) = x^2 + x$ determine each of the following.

a) $f(g(2))$

$4 - (x^2 + x)$
$= 4 - (2^2 + 2)$
$= \boxed{-2}$

b) $g(f(-3))$

$(4-x)^2 + (4-x)$
$= (4-(-3))^2 + (4-(-3))$
$= 49 + 7$
$= \boxed{56}$

c) $g(g(1))$

$g(1) = 2$
$2^2 + 2 =$
$\boxed{6}$

d) $g(g(g(1)))$

$g(1) = 2$
$2^2 + 2 = 6$
$6^2 + 6 = \boxed{42}$

Example

If $f(x) = x - 3$ and $g(x) = \sqrt{x}$ determine each of the following.

Put g in f →

a) $(f \circ g)(x) = \sqrt{x} - 3$

Put f in g →

b) $(g \circ f)(x) = \sqrt{x - 3}$

c) Does the order matter when composing functions? **Yes**

d) State the domain of $f(x)$, $g(x)$, $(f \circ g)(x)$, and $(g \circ f)(x)$.

$f(x): x \in \mathbb{R}$

$g(x): x \geq 0$
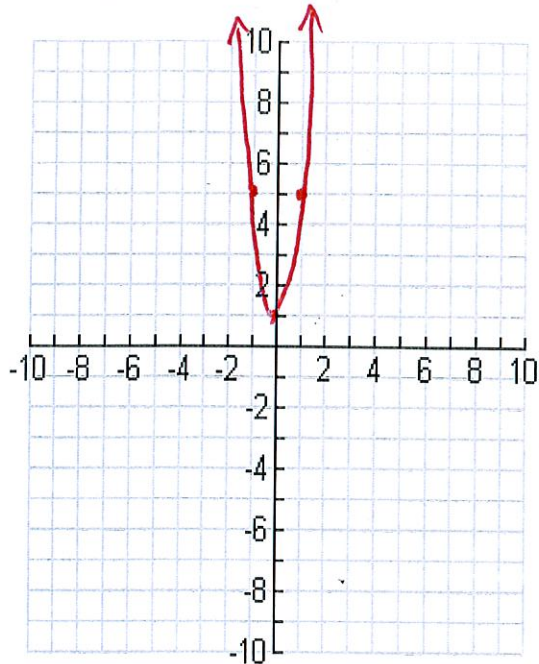
$(f \circ g)(x): x \geq 0$

$(g \circ f)(x): x \geq 3$

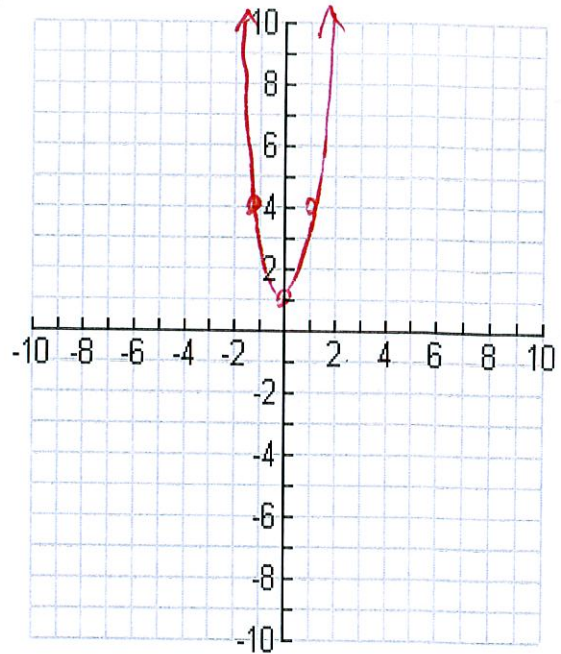**Example** Use your graphing calculator to help you graph.

If $f(x) = x^2 + 1$ and $g(x) = 2x$. Determine the equation of each composite function, graph it and state its domain and range.

a) $y = f(g(x))$

b) $y = f(f(x))$

$$y = (2x)^2 + 1$$

$$y = 4x^2 + 1$$

$$\boxed{\begin{array}{l} \underline{D}: X \in \mathbb{R} \\ \underline{R}: y \geq 1 \end{array}}$$

$$y = (x^2 + 1)^2$$

$$y = (x^2 + 1)(x^2 + 1)$$

$$y = x^4 + 2x^2 + 1$$

$$\boxed{\begin{array}{l} \underline{D}: X \in \mathbb{R} \\ \underline{R}: y \geq 1 \end{array}}$$

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Example

Determine two functions, $f(x)$ and $g(x)$ where $h(x) = f(g(x))$.

a) $h(x) = \sqrt{3x-2}$

"Outer" function is $\left(\sqrt{x}\right)$

"Inner" function is $\left(3x-2\right)$

$$f(x) = \sqrt{x}$$
$$g(x) = 3x-2$$

b) $y = (x-4)^2 + 3(x-4) + 4$

"Outer" function is $\left(x^2 + 3x + 4\right)$

"Inner" function is $\left(x-4\right)$

$$f(x) = x^2 + 3x + 4$$
$$g(x) = x-4$$

Textbook: page 507 questions 1 to 12