Key

# MATH 30-1
# CHAPTER 4
# TRIG
# &
# THE UNIT CIRCLE

$$\theta = \frac{a}{r}$$

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{1}{\tan x} \qquad \cot x = \frac{\cos x}{\sin x}$$

$$\csc x = \frac{1}{\sin x} \qquad \sec x = \frac{1}{\cos x}$$

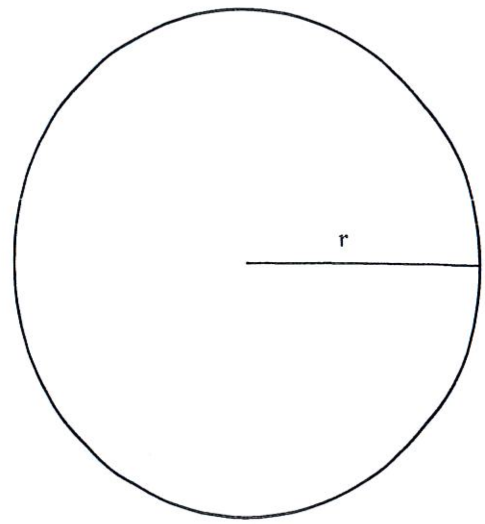
**Math 30-1**
**Unit:** **Trigonometry and The Unit Circle**
**Topic:** **Angles and Angle Measure**

Objectives:

- Sketch angles in standard position measured in degrees and radians.
- Convert angles in degree measure to radian measure and vice versa
- Determine the measure of angles that are coterminal with a given angle

Radian - a "unit" (like degrees) used to measure the size of an angle

- It is a ratio of the length of the arc subtending an angle and the length of the radius of the circle
- $radian = \dfrac{length\ of\ arc\ \ subtended\ by\ angle}{radius}$




Determine the size of a $360^o$ angle in radians

Length of arc: circumference
$= 2\pi r$

$radian = \dfrac{2\pi r}{r}$

$\boxed{radian = 2\pi}$

| **Degrees** | $360^o$ | $180^o$ | $90^o$ | $270^o$ | $30^o$ | $60^o$ | $45^o$ |
|---|---|---|---|---|---|---|---|
| **Radians As Exact Values** | $2\pi$ | $\pi$ | $\frac{\pi}{2}$ | $\frac{3\pi}{2}$ | $\frac{\pi}{6}$ | $\frac{\pi}{3}$ | $\frac{\pi}{4}$ |
| **Radians as an Approximate Value** | 6.28 | 3.14 | 1.57 | 4.71 | 0.52 | 1.05 | 0.79 |

**Example**

| Angle in Degrees | Radians as EXACT values | Radians as APPROXIMATE values |
|---|---|---|
| $135^o$ $90+45$ | $\frac{\pi}{2}+\frac{\pi}{4}=\frac{3\pi}{4}$ | 2.36 |
| $-120^o$ | $-\frac{2\pi}{3}$ | $-2.10$ |
| $300^o$ $270+30$ | $\frac{3\pi}{2}+\frac{\pi}{6}=\frac{10\pi}{6}$ | 5.24 |
| $x^o$ | $\frac{x\pi}{180}$ | |

To convert, you can multiply by $\frac{\pi}{180}$

**Example**

| Angle in Radians | Degrees |
|---|---|
| $\frac{4\pi}{3}$ $\pi+\frac{\pi}{3}$ | $180+60=240^\circ$ |
| 2.57 | $2.57\times\frac{180}{\pi}=147^\circ$ |
| $\frac{11\pi}{6}$ $2\pi-\frac{\pi}{6}$ | $360-30=330^\circ$ |
| 8 | $8\times\frac{180}{\pi}=458^\circ$ |

To convert, you can multiply by $\frac{180}{\pi}$

| How to Convert from Degrees to Radians | | |
|---|---|---|
| | As Exact Values | $\times\frac{\pi}{180}$ and reduce |
| | As Rounded Values | $\times\frac{\pi}{180}$ |

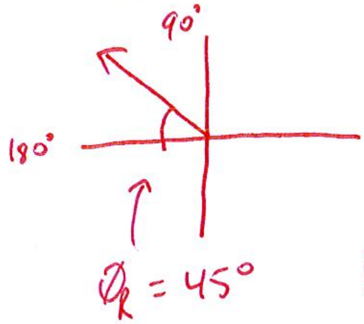
| | | |
|---|---|---|
| **How to convert from Radians to Degrees** | **If Radians are given as Exact Value or Rounded value** | $\times \frac{180}{\pi}$ |

**Summarize**
**Recall**
Reference Angle

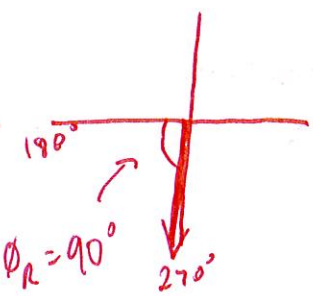
- $0^o \le \theta_R \le 90^o$ OR $0 \le \theta_R \le \frac{\pi}{2}$
- The number of degrees a terminal arm of an angle is **<u>above or below the nearest $x$ - axis</u>**

**Example**
Sketch the following angles in standard position **<u>and</u>** determine the reference angle.

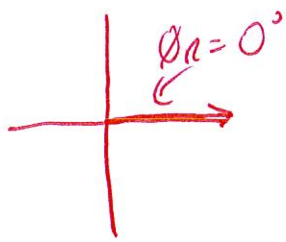
a. $135^o$



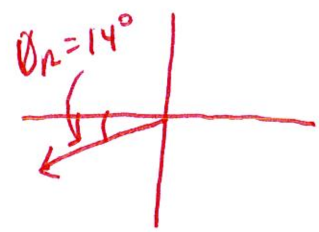

b. $270^o$



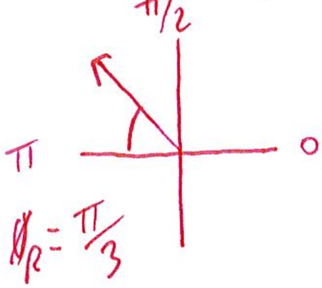

c. $360^o$



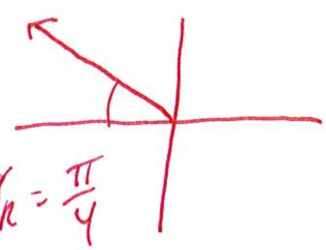

d. $-166^o$



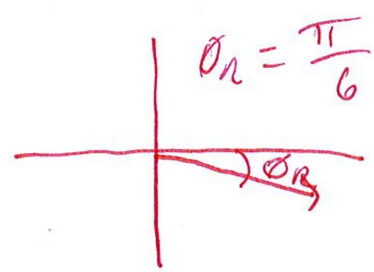

e. $\frac{2\pi}{3}$




f. $-\frac{5\pi}{4}$




g. $\frac{11\pi}{6}$



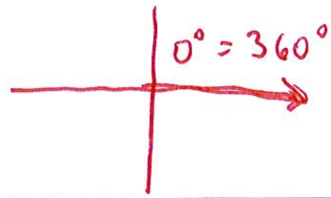
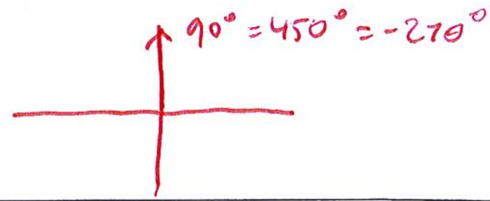
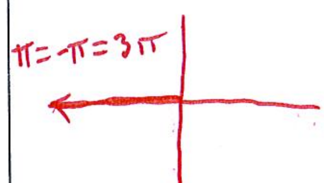

h. $\frac{3\pi}{2}$

i. $\frac{13\pi}{8}$

j. 5.25

**Coterminal Angles**

- angles, when drawn in standard position, have the same terminal arms
- Example of coterminals are:

| **Coterminal Angles** | **Sketch** |
|---|---|
| $0^o$ and $360^o$ |   |
| $90^o$ and $450^o$ and $-270^o$ |   |
| $\pi$ and $-\pi$ and $3\pi$ |   |

**Example**

- State 2 angles coterminal to $50^o$ that are positive

$50 + 360 = 410^\circ$

$410 + 360 = 770^\circ$

- State 2 angles coterminal to $50^o$ that are negative

$50 - 360 = -310^\circ$

$-310 - 360 = -670^\circ$

- Summarize all angles that are coterminal to $50^o$ using a **"general statement"**

$50 + 360n, \quad n \in I$

**Example**
Identify the angles that are coterminal to $-435^o$ that satisfy $0^o \le \theta \le 360^0$.

$-435 + 360 + 360 = \boxed{285^o}$

**Example**
Identify the angles that are coterminal to $\frac{2\pi}{3}$ that satisfy $-2\pi \le \theta \le 2\pi$.

$\frac{2\pi}{3} - 2\pi \rightarrow \frac{2\pi}{3} - \frac{6\pi}{3} = \boxed{-\frac{4\pi}{3}}$ and $\boxed{\frac{2\pi}{3}}$

**Example**
Are the angles $185^o$ and $-545^o$ coterminal?

$185 - 360 = -175$

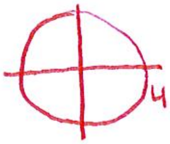
$-175 - 360 = -535$ no, not coterminal

$-535 - 360 = -895$

**Textbook** Page 175 #2 - 9, 11

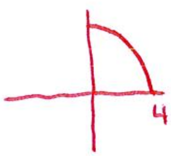
**Math 30-1**
**Unit:** **Trigonometry and The Unit Circle**
**Topic:** **Arc Lengths**
Objectives:
- Solve problems involving arc length, central angles and the radius in a circle

***Arc Lengths***
Investigation
- Consider a circle on a coordinate grid whose center is at the origin and radius is 4 cm. Estimate the circumference of that circle.



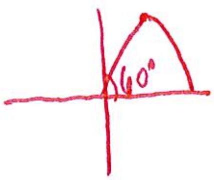

$4 \times 2\pi = 8\pi$

- Suppose a right angle is drawn within that circle with its center at the origin and whose rays terminate on the circumference of the circle. Estimate the length of the arc being subtended by that angle.




$8\pi \div 4 = 2\pi$ (1)

- Suppose a triangle with an angle of $60^{\circ}$ at the origin is drawn within that circle and whose rays terminate on the circumference of the circle. Estimate the length of the arc being subtended by that angle.

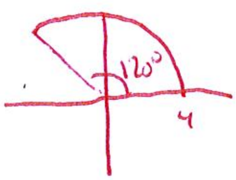



(2)

$\frac{2}{3}$ of the last one.

$\frac{2}{3} \cdot 2\pi = \frac{4\pi}{3}$

- Suppose a triangle with an angle of $120^{\circ}$ at the origin is drawn within that circle and whose rays terminate on the circumference of the circle. Estimate the length of the arc being subtended by that angle.




(3)

Double the last one.

$= \frac{8\pi}{3}$

- What do you notice about the ratio of the length of the arc to the circumference of the circle AND the ratio of the angle at the origin and one complete revolution?

For (1), $\frac{2\pi}{8\pi} = \frac{1}{4}$, $\frac{90}{360} = \frac{1}{4}$ Same

(2) $\frac{4\pi/3}{8\pi} = \frac{4\pi}{24\pi} = \frac{1}{4}$ $\frac{60}{360} = \frac{1}{6}$ Same

You get the idea. They're the same.

Notice that the length of the arc is proportional to the radius of the circle. The ratio of the arc length to the circumference of the circle should be the same as the ratio of the central angle to one complete rotation. Show that relationship using a proportion. Simplify.

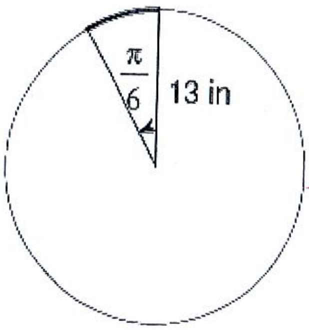
| **When central angle is in degrees** | **When central angle is in radians** |
|---|---|
| $\frac{a}{2\pi r} = \frac{\emptyset}{360}$ <br> $a = \frac{\emptyset \cdot 2\pi r}{360}$ | $\frac{a}{2\pi r} = \frac{\emptyset}{2\pi}$ <br> $a = \frac{\emptyset \cdot \cancel{2\pi} r}{\cancel{2\pi}}$ <br> $a = \emptyset r$ |

Complete the chart.

| Arc Length | Central angle | Radius |
|---|---|---|
| $a = \frac{120 \cdot 2\pi \cdot 5}{360}$ <br> $\boxed{a = 10.47\text{ cm}}$ | $120^o$ <br> degrees | 5 *cm* |
| 2*cm* | $\frac{7\pi}{6}$ <br> radians | $2 = \frac{7\pi}{6} \cdot r$ <br> $12 = 7\pi r$ <br> $\boxed{r = 0.55\text{ cm}}$ |
| 3 *cm* <br> Default to radians | $3 = \emptyset \cdot 3$ <br> $\boxed{\emptyset = 1}$ <br> radians | 3 *cm* |

**Example**

Determine the length of the arc in the diagram below.



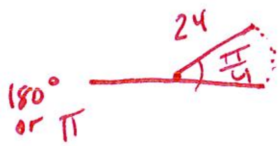

$a = \theta r$

$a = \frac{\pi}{6} \cdot 13$

$\boxed{a = 6.8 \text{ in}}$

**Example**

The windsheild wiper on a car is 24 inches long. How many inches will the tip of the wiper trace out in $\frac{1}{4}$ of a revolution? Depends on window. Bad question. Assume 180° revolution.

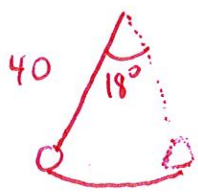



$a = \frac{\pi}{4} \cdot 24$

$\boxed{a = 18.85 \text{ in}}$

**Example**

A 40-inch pendulum swings through an angle of $18^o$. Find the length of the arc, in inches, through which the tip of the pendulum swings.




$a = \frac{\theta \cdot 2\pi r}{360}$

$a = \frac{18 \cdot 2 \cdot \pi \cdot 40}{360}$

$\boxed{a = 12.57 \text{ in}}$

**Example**

An arc DE of a circle, center O, is $\frac{1}{6}$ of the circumference. Determine the size of $\angle DOE$ to the nearest one hundredth of a radian. This means the angle is $\frac{1}{6}$ of $2\pi$.

$\frac{1}{6} \cdot 2\pi = \frac{\pi}{3} = \theta$

$\boxed{\theta = 1.05}$

**Example**

arc length

A person on a ferris wheel moves a distance of 5 meters from position P to position Q. If the diameter of the wheel is 18 meters, determine the measure of the central angle to the nearest tenth of a degree.

radius is 9

$a = \theta r$

$5 = \theta \cdot 9$

$\theta = \frac{5}{9}$ ← this is in radians. Change to degrees.

$\frac{5}{9} \times \frac{180}{\pi} = \boxed{31.8^\circ}$

**Example**

A satellite makes one complete revolution of the earth in 90 minutes. Assume that the orbit is circular and that the satellite is situated 280 km above the equator. If the radius of the earth at the equator is 6400 km, then the speed of the satellite, in km/h, to the nearest hundredth is . . . .

Total orbit radius: $6400 + 280 = 6680$ km

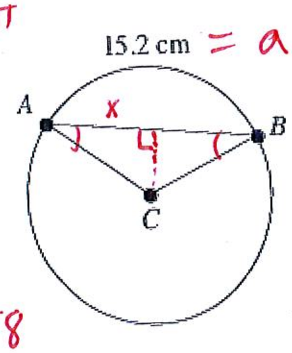
90 minutes = 1.5 hours

$a = \theta r = 2\pi(6680) = 41971.68$ km

Velocity $= \frac{d}{t} \rightarrow = \frac{41971.68}{1.5} = \boxed{27981.12 \text{ km/h}}$

**Example**

A circle with center C and minor arc AB measuring 15.2 cm is shown. If $\angle ABC = \angle BAC = \frac{\pi}{6}$ find the perimeter of the triangle ABC to the nearest tenth.




Angles in a triangle add up to $360^\circ = 2\pi$

$\frac{\pi}{6} + \frac{\pi}{6} = \frac{\pi}{3}$

This means $\angle ACB = \frac{2\pi}{3}$

$15.2 = \frac{2\pi}{3} \cdot r \rightarrow r = 7.26$

$\cos\frac{\pi}{6} = \frac{x}{7.26} \rightarrow x = 6.29 \rightarrow$ That side $= 12.58$

Total: $12.58 + 7.26 + 7.26$

$= \boxed{27.1}$ cm

**Textbook** Page 175 #12, 13, 14a

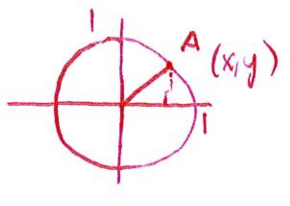
**Math 30-1**
**Unit:** **Trigonometry and The Unit Circle**
**Topic:** **The Unit Circle**

Objectives:

- Generalize the equation of the unit circle with center $(0,0)$ and radius $r$
- Locate the coordinates of points on the unit circle

The Unit Circle

- Draw a circle whose center is at the origin of a coordinate grid and whose radius is 1 unit long – this is called a unit circle




Quadrantals:
(1,0)
(0,1)
(-1,0)
(0,-1)

- Write the coordinates of the four points that intersect the $x$ and $y$ - axis with the circle. These points are called "quadrantals" . . . we will investigate them later
- Make a point on the unit circle in quadrant I. Call it point A and give it the coordinates $(x, y)$. Using the origin and point A, create a right-angled triangle with the third point being on the positive $x$-axis.
- Using Pythagorean Theorem, determine the equation of the unit circle

$x^2+y^2=1^2$
$y^2=1-x^2$
$y=\sqrt{1-x^2}$

- Consider how the equation of the circle might change if the radius was altered . . .

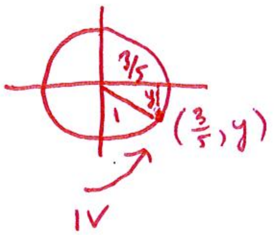
| Center of circle | Radius of circle | Equation of circle |
|---|---|---|
| $(0,0)$ | 4 units | $y=\sqrt{16-x^2}$ |
| $(5,3)$ | $\sqrt{2}$ units | $y=\sqrt{2-(x-5)^2}+3$ |
| $(-2,-8)$ | 5 units | $y=\sqrt{25-(x+2)^2}-8$ |
| $(0,-1)$ | $\frac{1}{4}$ units | $y=\sqrt{\frac{1}{16}-x^2}-1$ |
| $(h,k)$ | $r$ units | $y=\sqrt{r^2-(x-h)^2}+k$ |

## Determining Coordinates of Points on the Unit Circle

Using the equation of the unit circle, if given either the $x$ or $y$-coordinate of a point, the missing value(s) can be found.

**Example**

Determine the coordinates of the point on the unit circle whose $x$-coordinate is $\frac{3}{5}$ and the point is in quadrant IV.




$$\left(\frac{3}{5}\right)^2 + y^2 = 1^2$$

$$y = \sqrt{1 - \left(\frac{3}{5}\right)^2}$$

$$y = -0.8 \longrightarrow \boxed{\left(\frac{3}{5}, -0.8\right)}$$

**Example**

Determine the coordinates of the points on the unit circle whose $y$-coordinate is $-\frac{\sqrt{3}}{2}$.

$$x^2 + \left(-\frac{\sqrt{3}}{2}\right)^2 = 1^2$$

$$x^2 + \frac{3}{4} = 1$$

$$x^2 = 0.25$$

$$x = \pm 0.5$$

$$\boxed{\begin{array}{c}\left(0.5, \frac{-\sqrt{3}}{2}\right) \\ \left(-0.5, \frac{-\sqrt{3}}{2}\right)\end{array}}$$

**Example**

Is the point $\left(-\frac{1}{2}, \frac{1}{2}\right)$ on the unit circle? How do you know?

$$\left(\frac{-1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = 0.5$$

If it were on the unit circle, it would equal 1.

$\boxed{\text{No.}}$

## Arc Length and the Unit Circle

- Recall the formula to determine the length of an arc being subtended by a central angle measured in radian units

$$a = \theta r$$

- If the radius of the circle is 1 unit, what can we conclude about the length of the arc being subtended by the central angle?

Same as the angle in radians.

**Example**

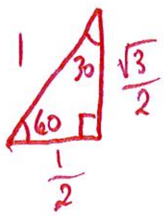
Determine the length of the arc subtended by an angle of $\frac{\pi}{2}$ on the unit circle.

$$a = \left(\frac{\pi}{2}\right)(1)$$

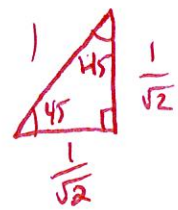
$$\boxed{a = \frac{\pi}{2}}$$

**Recall: Special Triangles** Unlike in Math 20-1, let's set the hypotenuse to 1. It used to be 2 and $\sqrt{2}$.

$30^o - 60^o - 90^o$ triangle




$45^o - 45^o - 90^o$ triangle




These are not given to you!

**What is the "purpose" of the special triangles?**

To find exact values for x and y components of common angles on the unit circle.

**Example**

Find the values of the following:

a. $\cos 30^o$ as an approximate value, rounded to 2 decimal places using your calculator

$0.87$

$\cos 30^o$ as an exact value using your special triangle

$\frac{\sqrt{3}}{2}$

b. $\sin 45^o$ as an approximate value, rounded to 2 decimal places using your calculator

$0.71$

$\sin 45^o$ as an exact value using your special triangle.

$\frac{1}{\sqrt{2}}$

**Recall: CAST RULE**

| | |
|---|---|
| S | A |
| T | C |

This tells you where a trig ratio is positive. "A" means all are positive there.

May be useful to note: special triangles work for radians, too.

$30^o = \frac{\pi}{6}$

$45^o = \frac{\pi}{4}$

$60^o = \frac{\pi}{3}$

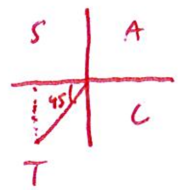
**Questions involving EXACT VALUES that require you to use your SPECIAL TRIANGLES & CAST RULE . . .**

**Determine** the exact value of the following.

(a) $\tan 60^\circ$

$$\tan 60 = \frac{\sqrt{3}}{2} \div \frac{1}{2}$$
$$= \frac{\sqrt{3}}{2} \cdot \frac{2}{1}$$
$$= \boxed{\sqrt{3}}$$

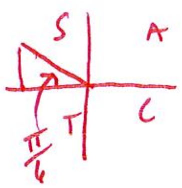
(b) $\tan 225^\circ$




$$= \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}$$
$$= \boxed{1}$$

(c) $\sin 225^\circ$

(d) $\cos\dfrac{5\pi}{6}$




$$= \frac{\frac{\sqrt{3}}{2}}{1}$$
$$\rightarrow = \boxed{\frac{-\sqrt{3}}{2}}$$

(e) $\tan\dfrac{4\pi}{3}$

(f) $\tan\dfrac{5\pi}{6}$

(g) $\sin\dfrac{7\pi}{4}$

(h) $\tan\left(-\dfrac{\pi}{4}\right)$

(i) $\tan\dfrac{7\pi}{4}$

(j) $\cos\left(-\dfrac{2\pi}{3}\right)$

(k) $\cos\dfrac{8\pi}{3}$

(l) $\sin^2\dfrac{5\pi}{3}$

**Worksheet**

Page 186 #1 – 6

**Math 30-1**
**Unit:** **Trigonometry and The Unit Circle**
**Topic:** **The Unit Circle**

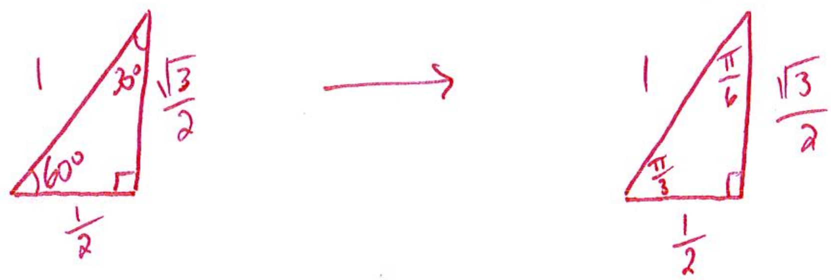
Objectives:

- Develop and apply the equation of the unit circle

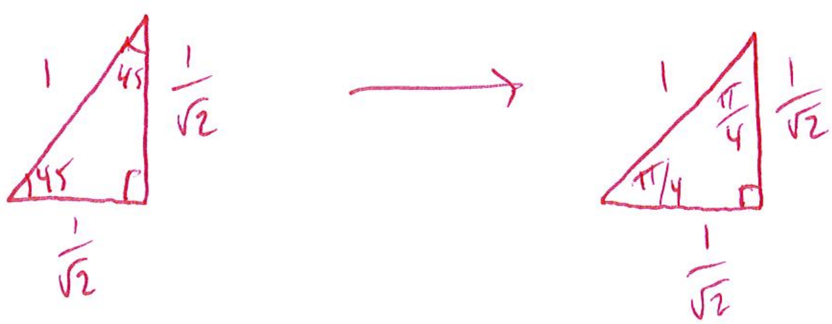
**<u>Points on the Unit Circle whose Terminal Arm is Multiples of</u>** $\frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{4}, \textit{and } \frac{\pi}{2}$

Use your special triangles to make similar triangles but whose hypotenuses are of length 1 unit

<u>30-60-90</u>

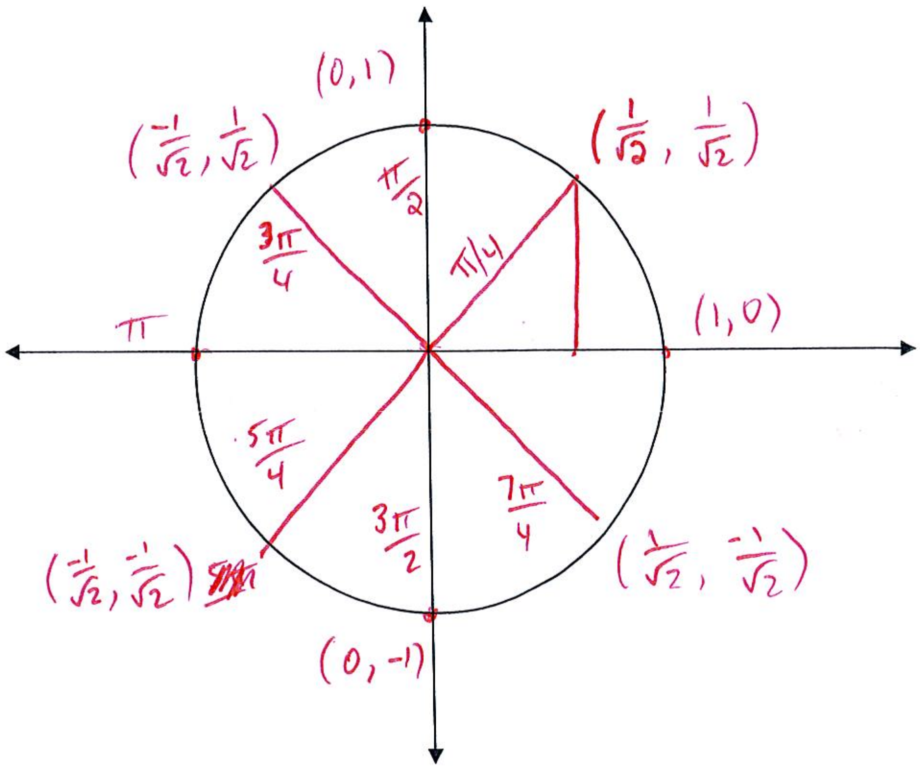



<u>45-45-90</u>



On the diagram of the unit circle,

- Draw the similar triangle you created for the $45^o - 45^o - 90^o$ triangle on the previous page, in quadrant I
- Write the coordinates of it's vertex that makes contact with the unit circle
- Repeat this process for all the points that are multiples of $\frac{\pi}{4}$ in the interval $0 \le \theta \le 2\pi$.




- Using your special triagnles, determine the values for $\sin 45^\circ$ and $\cos 45^\circ$.

$\sin 45 = \frac{1}{\sqrt{2}}$

$\cos 45 = \frac{1}{\sqrt{2}}$

- Compare the values you just found to the coordinates you've written on the unit circle corresponding to points on the terminal arm of angles whose reference angles are $45^\circ$. Make a generalization.

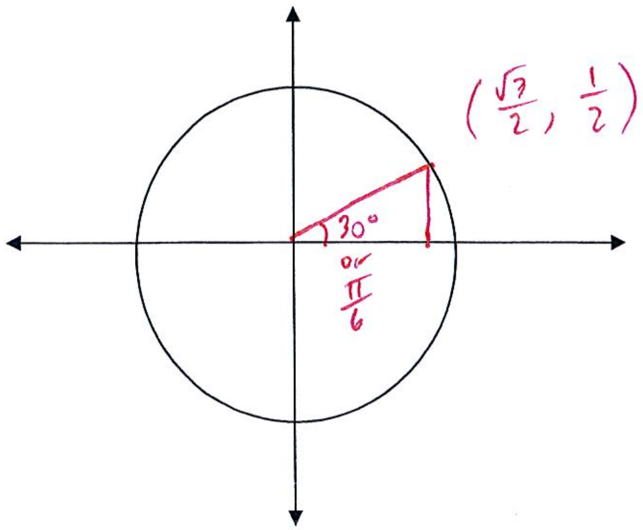
Combine with CAST rule.

$\sin \theta_R$ gives you $y$.

$\cos \theta_R$ gives you $x$.

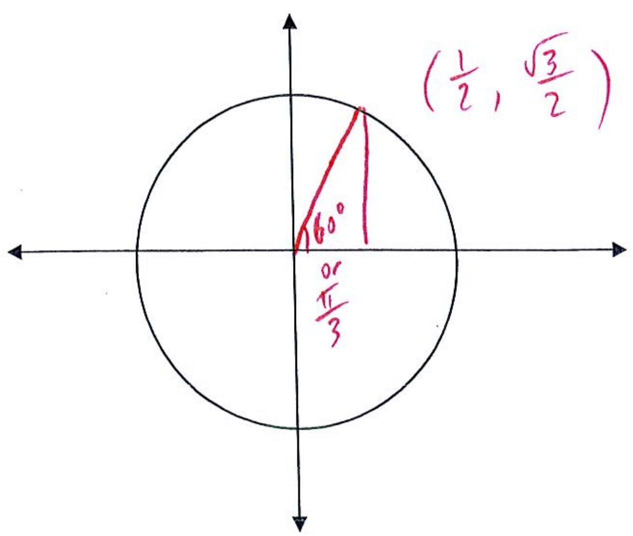
On the diagram of the unit circle,

- Draw the similar triangle you created for the $30^o - 60^o - 90^o$ triangle on the previous page in quadrant I . . . put the $30^o$ at the origin. Write the coordinates of the point on the triangle that that makes contact with the unit circle



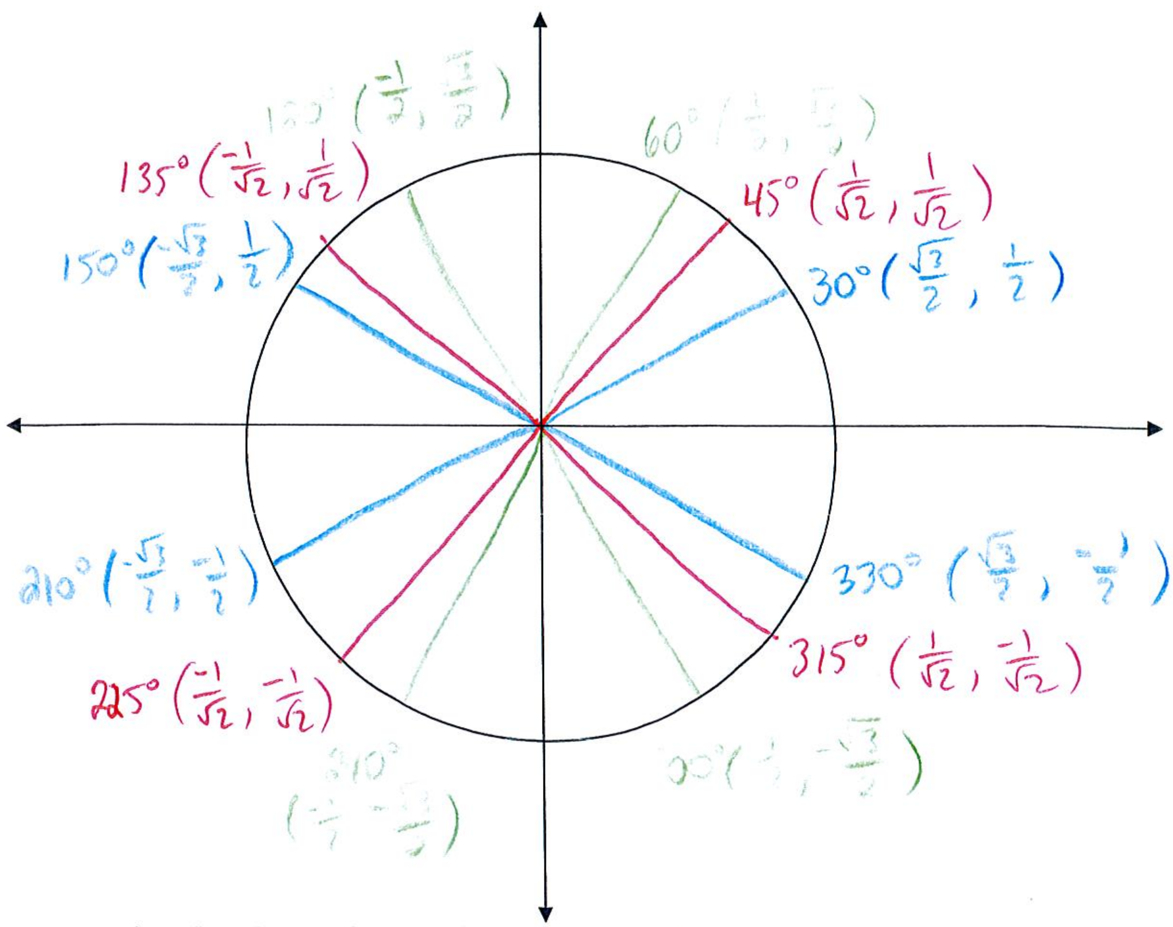

On the diagram of the unit circle,

- Draw the similar triangle you created for the $30^o - 60^o - 90^o$ triangle on the previous page in quadrant I . . . put the $60^o$ at the origin. Write the coordinates of the point on the triangle that that makes contact with the unit circle



**Summary**

On the unit circle, label every coordinate that is the intersection point between the unit circle and a terminal arm whose reference angle is 30°, 45°, or 60°.




**Summarize what the *x* and *y* coordinate of each coordinate on the unit circle are indicative of.**

x: cos of the reference angle (adj)

y: sin of the reference angle (opp)

**Example**

If $P(\theta)$ is the point at the intersection of the terminal arm of an angle $\theta$ and the unit circle, determine the **exact coordinates** of each of the following:

$\theta_r = \frac{\pi}{4}$ QII: a) $P\left(\frac{3\pi}{4}\right)$ $\left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$\theta_r = \frac{\pi}{6}$ QIV b) $P\left(-\frac{\pi}{6}\right)$ $\left(\frac{\sqrt{3}}{2}, \frac{-1}{2}\right)$

c) $P\left(\frac{5\pi}{3}\right)$ $\theta_r = \frac{\pi}{3}$ QIV $\left(\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$

**Example**

Determine one positive and one negative measure for $\theta$ if $P(\theta) = \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ QII

adj opp

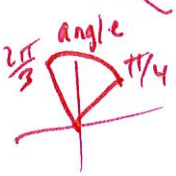
$\theta_r = 30^\circ$

$\rightarrow \boxed{\theta = 150^\circ}$ or $\boxed{-210^\circ}$

**Example** Q1 unfamiliar $\rightarrow \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

Q2 $A\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ and $B\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ are two points on the unit circle. If an object rotates counterclockwise from Point A to Point B through what angle has it rotated? Express your answer in radians as an exact value.

$\theta_r = \frac{\pi}{3} \rightarrow \theta = \frac{2\pi}{3}$ $\theta = \frac{\pi}{4}$




$\frac{2\pi}{3} - \frac{\pi}{4} = \frac{8\pi}{12} - \frac{3\pi}{12} = \boxed{\frac{5\pi}{12}}$

**Example** cos sin Q2

The point $T(-0.8829, 0.4695)$ lies on the unit circle. Determine the value of $\theta$ where $\theta$ is the anlge made by the positive $x$-axis and the line passing through $T$.

$\cos^{-1}(-0.8829) = 152^\circ$

**Special Triangles vs Unit Circle**

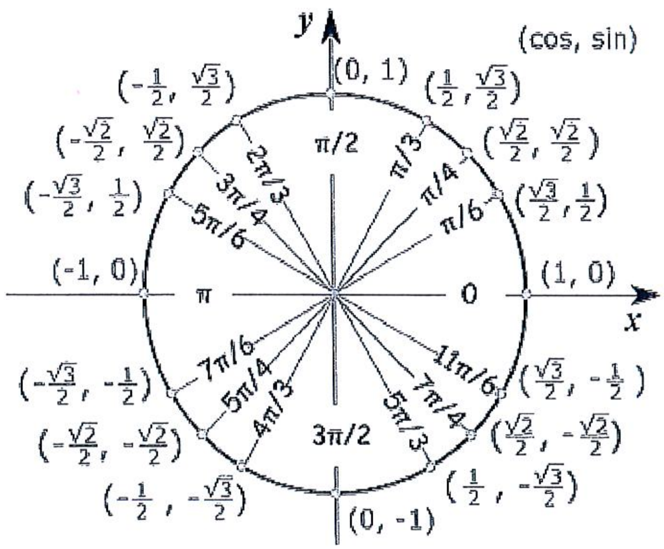
- Both serve the same purpose . . . and that is to determine EXACT VALUES of trig ratios of specific angles ( $30^o$ , $45^o$ , $60^o$ as well as $0^o$ , $90^o$ , $180^o$, $270^o$, & $360^o$ )
- What is one "limit" of the unit circle with regards to finding a trig ratio? How do we deal with that?

  The radius of 1. Any other radius, multiply spec. triangle side lengths accordingly.
- What is one "limit" of using special triangles?

  Only works for 30, 45, 60 angles.

**Summary**

- The equation for the unit circle is $x^2 + y^2 = 1$. It can be used to determine if a point is on the unit circle or to find one coordinate, given the other. For a radius of $r$ , the equation is $x^2 + y^2 = r^2$
- On the unit circle, the measure in radians of the central angle and the arc subtended by that angle are numermically the same
- Special triangles can be used to find coordiantes of points on unit circle
- The $x$-coordinate of every point on the unit circle represents the cosine ratio of that angle
- The $y$-coordinate of every point on the unit circle represents the sine ratio of that angle
- To find the tangent ratio of an angle, divide the $y$-coordinate (sine ratio) by the $x$-coordinate (cosine ratio)




**Textbook** Page 186 #1-6

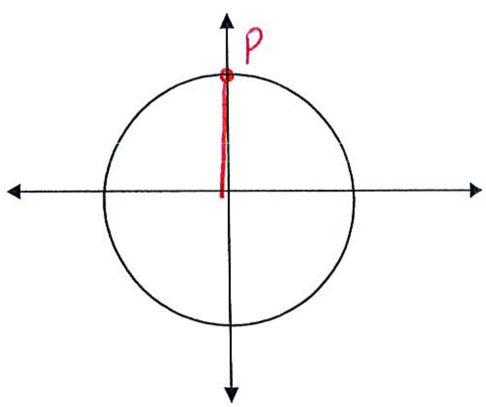
**Math 30-1**
**Unit:** **Trigonometry and The Unit Circle**
**Topic:** **Trig Ratios**

Objectives:

- Relate trig ratios to the coordinates of points on the unit circle
- Determine exact and approximate values for trig ratios
- Identify the measure of angles that generate specific trig values
- Solve problems involving trig ratios

**Investigation – Trig Ratios and the Unit Circle**

On the unit circle below, draw a positive angle $\theta$ in standard position that terminates on the positive y-axis. Label the point of intersection between the terminal arm of that angle and the unit circle *P*.




What are the coordinates of that point? (0, 1)

What does the $x$-coordinate of that point have to do with a $90^o$ angle? Cos 90

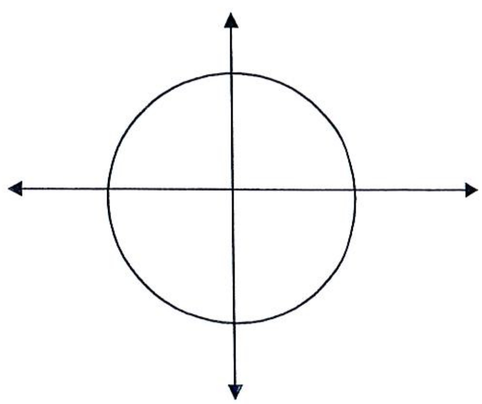
What does the $y$-coordinate of that point have to do with a $90^o$ angle? Sin 90

What if the angle $\theta$ in standard position terminated on the negative x-axis?

- What is the measure of $\theta$ in standard position? $180^o$
- What are the coordinates of that point? (−1, 0)
- What is $\cos 180^o$ ? −1
- What is $\sin 180^o$ ? 0

**Complete the following table involving all the points of intersection between the unit circle and the $x$ and $y$ axis.**

| **Quadrantal** | **Coordinates of point on unit circle** | $\sin\theta$ | $\cos\theta$ | $\tan\theta$ |
|---|---|---|---|---|
| $0^0$ | (1,0) | 0 | 1 | 0 |
| $90^0$ | (0, 1) | 1 | 0 | Error |
| $180^0$ | (-1,0) | 0 | -1 | 0 |
| $270^0$ | (0 ,-1) | -1 | 0 | Error |
| $360^0$ | (1,0) | 0 | 1 | 0 |



Note that $\tan\theta = \frac{\sin\theta}{\cos\theta}$

What appears to be the maximum and minimum values for $\cos\theta$ and $\sin\theta$? Use words and an inequality to express your answer. Verify your answer using a calculator and find the sine and cosine of angles of various sizes. Both are between -1 and 1.

$-1 \leq \cos\theta \leq 1$   $-1 \leq \sin\theta \leq 1$

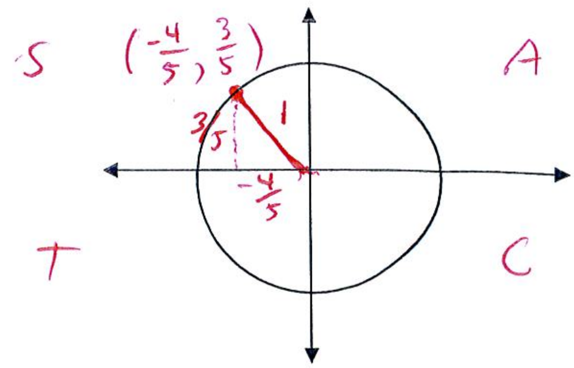
We can see that $\tan 90^0$ is undefined. What does this look like on your calculator?

Error.

## Example

The point $Q\left(-\frac{4}{5},\frac{3}{5}\right)$ is on the intersection point between the unit circle and the terminal arm of an angle $\theta$ in standard position.

Determine the exact values of the 3 primary trig ratios for $\theta$.




$\sin\theta = \frac{\frac{3}{5}}{1} = \frac{3}{5}$

$\cos\theta = \frac{\frac{-4}{5}}{1} = \frac{-4}{5}$

$\tan\theta = \frac{3/5}{-4/5} = \frac{3}{5}\cdot\frac{5}{-4}$

$= -\frac{3}{4}$

**Reciprocal Trig Ratios**

Recall: the reciprocal of a number is simply $\frac{1}{number}$

There are reciprocals for each of the 3 primary trig ratios . . .

All on your formula sheet

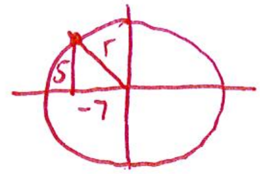
| Primary trig ratio | Name of reciprocal ratio | Abbreviation | ratio |
|---|---|---|---|
| Sine | Cosecant | csc | $\frac{hyp}{opp}$ |
| Cosine | Secant | sec | $\frac{hyp}{adj}$ |
| tangent | cotangent | cot | $\frac{adj}{opp}$ |

Referring to the example at the top of the page, determine the three reciprocal ratios for the angle $\theta$ whose terminal arm passes through point $Q$.

$\csc\theta = \frac{5}{3}$

$\cot\theta = \frac{-4}{3}$

$\sec\theta = \frac{-5}{4}$




**Example** Not a unit circle

The point (-7, 5) is on the terminal arm of angle $\theta$ in standard position. What is the exact value for all six trig ratios for $\theta$?

$$5^2 + (-7)^2 = r^2$$

$$74 = r^2 \rightarrow r = \sqrt{74}$$

$$\sin\theta = \frac{5}{\sqrt{74}} \qquad \csc\theta = \frac{\sqrt{74}}{5}$$

$$\cos\theta = \frac{-7}{\sqrt{74}} \qquad \sec\theta = \frac{-\sqrt{74}}{7}$$

$$\tan\theta = -\frac{5}{7} \qquad \cot\theta = -\frac{7}{5}$$

**Example**

Angle $P$ has a terminal arm the first quadrant. Given that $\sec P = 2$, determine the value of $\sin P - \cos P$.

$$\sec P = 2 \longrightarrow \cos P = \frac{1}{2}$$

Special triangles: $\frac{1}{2}$ is adjacent

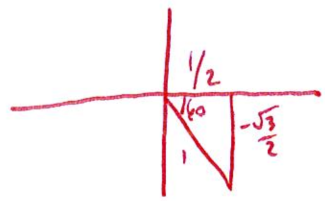
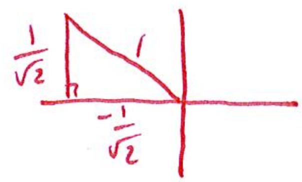
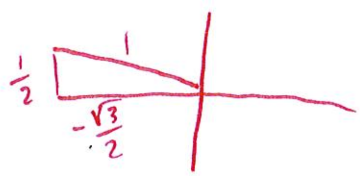
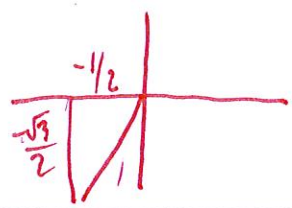
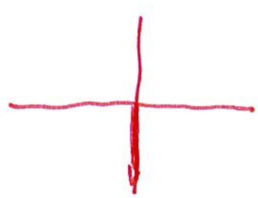
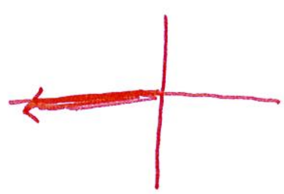
$$\rightarrow 60^\circ = \theta$$

$$\sin 60 = \frac{\sqrt{3}}{2}$$

$$\cos 60 = \frac{1}{2}$$

$$\therefore \sin P - \cos P = \frac{\sqrt{3}}{2} - \frac{1}{2} = \boxed{\frac{\sqrt{3}-1}{2}}$$

**Example**
Determine the exact value for each ratio. Include a diagram.

| Ratio | Reference Angle | Diagram | Answer |
|---|---|---|---|
| $\sec 300^\circ$<br>$\frac{hyp}{adj}$ | $60^\circ$ |   | $\frac{1}{1/2} = 2$ |
| $\cot(-225^\circ)$<br>$\frac{adj}{opp}$ | $45^\circ$ |   | $\frac{-1/\sqrt{2}}{1/\sqrt{2}} = -1$ |
| $\csc\frac{5\pi}{6}$<br>$\frac{hyp}{opp}$ | $\frac{\pi}{6}$ |   | $1/(1/2) = 2$ |
| $\sec\frac{4\pi}{3}$<br>$\frac{hyp}{adj}$ | $\frac{\pi}{3}$ |   | $1/-1/2 = -2$ |
| $\cot\frac{3\pi}{2}$<br>$\frac{adj}{opp}$ or $\frac{\cos\theta}{\sin\theta}$ | $\frac{\pi}{2}$ |  | $\frac{0}{-1} = 0$ |
| $\csc\pi$<br>$\frac{hyp}{opp}$ | $0$ |  | Undefined |

## Exact Values of Trig Ratios VS Approximate Values

### Approximate Values for Trig Ratios

There are times when you just need to know the "approximate" values of a ratio in which case a caluclator can be used.

The MODE that your calculator should be in depends on whether the angle given is in degrees or radians.

| Ratio | Mode | Answer |
|---|---|---|
| $\tan\frac{11\pi}{6}$ | Rad | $-0.5774$ |
| $\cos(-30^0)$ | Deg | $0.8660$ |
| $\sin 8.4$ | Rad | $0.8546$ |
| $\sec(25^o)$ | Deg | $\frac{1}{\cos 25} = 1.1034$ |

**Summary**

- Points on the intersection of the unit circle and the terminal arm of an angle in standard position can be given the coordinates $P(\theta)=(\cos\theta\,,\,\sin\theta)$
- Each primary trig ratio has a reciprocal trig ratio

$$\csc\theta=\frac{1}{\sin\theta} \qquad \sec\theta=\frac{1}{\cos\theta} \qquad \cot\theta=\frac{1}{\tan\theta}$$

- Exact values for trig ratios of special angles $\left(0,\frac{\pi}{6},\frac{\pi}{4},\frac{\pi}{3},\frac{\pi}{2}\right)$ and their multiples can be found using the points on the unit circle
- Approximate values of any trig ratio can be found using a calculator
- Given a tri ratio, $\theta$ can be found using the inverse of the ratio given and your knowledge of reference angles and the CAST rule
- Given the coordinates of a point on the terminal arm of an angle in standard position, you can find the trig ratios for that angle by drawing a right triangle and using pythagorean theorem to find the missing side

**Textbook** Page 201 #1-9, 13, 19

**Math 30-1**
**Unit:** **Trigonometry and The Unit Circle**
**Topic:** **Introduction to Trig Equations**

Objectives:

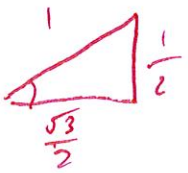
- Algebraically solve first trig equations in radians and degrees
- Verify a solution to a trig equation
- Identify exact and approximate solutions in a restricted domain
- Determine the general solution to a trig equation

**Approximate Values for Angles**

Given a trig ratio you can find the measure of the angle by reversing the process used above. To find $\theta$ you must know use the INVERSE trig function ($2^{nd}$ button on calculator following by whichever ratio you were given).

**Example**

Given that $\sin\theta = \frac{1}{2}$, determine the value of $\theta$ assuming it is in the first quadrant.




$\rightarrow 30°$

**Example**

If $\cot\theta = \sqrt{3}$, determine the value of $\theta$, given $0 \le \theta \le \frac{\pi}{2}$.

$\tan\theta = \frac{1}{\sqrt{3}}$ $\rightarrow$ the $\sqrt{3}$ hints that special triangles are involved

$\tan\theta = \frac{opp}{adj}$ $\rightarrow$ $\tan 30° = \frac{1/2}{\sqrt{3}/2}$ $\rightarrow$ $\frac{1}{2} \times \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\rightarrow \theta = 30°$

$\rightarrow \frac{\pi}{6}$



Alternatively, you could use $\tan^{-1}(1/\sqrt{3})$ to get $30°$

**Note:** When using $\sin^{-1}$ , $\cos^{-1}$ , $\tan^{-1}$ on your calculator, you wil always be given an ACUTE angle even though your answer(s) may be in other quadrants. Consider the answer you get from your calculator the REFERENCE angle and use your CAST RULE to determine all possible answers.

When you use inverse trig, plug in the positive equivalent of your number to get the reference angle.

S | A
T | C

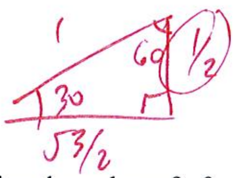
**Example** $\rightarrow -\frac{1}{2}$

If $\cos\theta = -0.5$ , determine the value of $\theta$ , given $0 \le \theta \le 2\pi$ .

cos is negative $\longrightarrow$ Q2 or Q3

cos is $\frac{adj}{hyp}$ $\longrightarrow$ special triangles

adj $\rightarrow$ $\theta = 60° = \frac{\pi}{3}$




$\boxed{Q2: \frac{2\pi}{3}}$ $\boxed{Q3: \frac{4\pi}{3}}$

**Example**

If $\tan\theta = 5.24$ , determine the value of $\theta$ , given $-180° \le \theta < 180°$ .

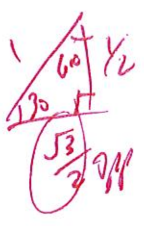
$\frac{opp}{adj}$ ~~[scribbled]~~ Q1 and Q3 $\rightarrow$ $\tan^{-1}(5.24) = \boxed{79.2°}$

$-180 + 79.2 = \boxed{-100.8°}$

**Example**

If $\csc\theta = \frac{2}{\sqrt{3}}$ , determine the value of $\theta$ , given $0 \le \theta \le 3.14$ . π radians

$\hookrightarrow \sin\theta = \frac{\sqrt{3}}{2} \longrightarrow \frac{opp}{hyp}$ Sin positive $\rightarrow$ Q1 or Q2




$\theta = 60° \rightarrow$ $\boxed{\frac{\pi}{3} \text{ Q1}}$ $\boxed{\frac{2\pi}{3} \text{ Q2}}$

**Example** Isolate the trig ratio.

Solve the equation $4\csc x - 8 = 0$ over the domain $[0, 360^0)$.

$4\csc x = 8$

$\csc x = 2$ special triangles $\rightarrow$ $\boxed{x = 30^\circ \; Q1}$ and $\boxed{x = 150^\circ \; Q2}$

$\hookrightarrow \sin x = \frac{1}{2}$

sin is positive in Q1 and Q2

A C T

**Example**

Solve the equation $7\cos\theta + 5 = 3 + 3\cos\theta$ , $0 \le \theta < 2\pi$ . Express answers as exact values.

$-3\cos\theta$ $-3\cos\theta$ radians

$4\cos\theta + 5 = 3$

$4\cos\theta = -2$

$\cos\theta = -1/2$

cos is negative in Q2 and Q3

$\theta_\ell = 60^\circ = \frac{\pi}{3}$

$\boxed{Q2: 120^\circ = \frac{2\pi}{3}}$

$Q3: 240^\circ$ $\boxed{= \frac{4\pi}{3}}$

**Note:** when asked to solve a trig equation, you may be asked to express your answer(s) as

- The solutions within a given domain, such as $0 \le \theta < 2\pi$ , or
- All possible solutions, expressed in general form
  - This is asking you for the every angle that is coterminal to the angles that are the solution within the domain $0 \le \theta < 2\pi$
  - It is usually written as $\theta + 2\pi n$ , $n \in I$

or $\theta + 360n$ if in degrees

**Example**

Detremine the general solution to the equation $17 + 3\cot\theta = 29$ in degrees, rounded to the nearest tenth.

$3\cot\theta = 12$

$\cot\theta = 4$

$\tan\theta = \frac{1}{4}$

Not a special triangle. Just use inverse tan.

$\rightarrow \boxed{\theta = 14^0 \; Q1}$

tan is positive in Q1 and Q3

$\boxed{\theta = 194^0 \; Q3}$

Need to use both for general solution

$\boxed{14 + 360n}$

$\boxed{194 + 360n}$

S | A
T | C

**Try**

1. Solve each equation for $-\pi \le \theta \le \pi$

a. $3\tan\theta - 3 = 5\tan\theta - 1$

$-3 = 2\tan\theta - 1$

$-2 = 2\tan\theta$

$-1 = \tan\theta \rightarrow$ Q2 or Q4

$\theta_R = \frac{\pi}{4}$ $\boxed{\theta = \frac{3\pi}{4}}$ $\boxed{\theta = \cancel{\frac{7\pi}{4}}}$ $\boxed{\frac{-\pi}{4}}$

b. $5(1+2\sin\theta) = 2\sin\theta + 1$

$5 + 10\sin\theta = 2\sin\theta + 1$

$8\sin\theta = -4$

$\sin\theta = \frac{-1}{2} \rightarrow$ Q3 and Q4

$\theta_R = \frac{\pi}{6} \rightarrow \boxed{\cancel{\frac{7\pi}{6}}}$ Q3 $\boxed{\frac{-5\pi}{6}}$

$\rightarrow \boxed{\cancel{\frac{11\pi}{6}}}$ Q4 $\boxed{\frac{-\pi}{6}}$

2. Solve each equation for $-180^{o} \le \theta \le 90^{o}$.

a. $2\csc\theta = 6$

$\csc\theta = 3$

$\sin\theta = \frac{1}{3}$ Q1 and Q2

$\boxed{\theta = 19.5^{\circ}}$ Q1 $\boxed{\cancel{\theta = 160.5^{\circ}}}$ Q2 outside domain

b. $-6 = 3\cot\theta$

$-2 = \cot\theta$

$\frac{-1}{2} = \tan\theta$ Q2 or Q4

$\boxed{\theta_R = 26.6^{\circ}} \rightarrow \boxed{\cancel{\theta = 153.4^{\circ}}}$

$\boxed{\theta = \cancel{333.4^{\circ}}}$

$\boxed{\theta = -26.6^{\circ}}$

3. Solve each equation for $-90^{o} \le \theta \le 180^{o}$.

a. $4\sec\theta = -5$

$\sec\theta = \frac{-5}{4}$

$\cos\theta = \frac{-4}{5}$ Q2 and Q3

$\theta_R = 36.9^{\circ}$ $\boxed{\theta = 143.1^{\circ}}$

$\boxed{\cancel{\theta = 216.9^{\circ}}}$ outside domain

b. $-\frac{1}{2} = \frac{1}{3}\csc\theta$

$\frac{-3}{2} = \csc\theta$

$\frac{-2}{3} = \sin\theta$ Q3 and Q4

$\theta_R = 41.8^{\circ}$

$\boxed{\cancel{\theta = 221.8^{\circ}}}$

$\boxed{\theta = \cancel{318.2}}$

$\boxed{\theta = -41.8^{\circ}}$

**Answers**

1. a) $\theta = -\frac{\pi}{4}, \frac{3\pi}{4}$ b) $\theta = -\frac{\pi}{6}, -\frac{5\pi}{6}$ 2 a) $\theta = \pm 71^{o}$ b) $\theta = -27^{o}$

3. a) $\theta = 143^{o}$ b) $\theta = -42^{o}$

**Textbook** Page 202 #10 – 12 & Page 211 #1 - 6

**Math 30-1**
**Unit:** **Trigonometry and The Unit Circle**
**Topic:** **More Trig Equations**

Objectives:

- Algebraically solve second degree trig equations in radians and degrees
- Verify a solution to a trig equation
- Identify exact and approximate solutions in a restricted domain
- Determine the general solution to a trig equation

**Recall:** Factor the following quadratic equations and solve for $x$.

Sum Product

$x^2+5x+4=0$

→ 4 and 1

→ $(x+4)(x+1)=0$

$\boxed{x=-4}$

$\boxed{x=-1}$

$\underbrace{x^2-1}=0$

Difference of squares

Square root both, multiply conjugates

→ $(x+1)(x-1)=0$

$\boxed{x=-1}$ $\boxed{x=1}$

$4x^2=9$

$4x^2-9=0$

Diff. of squares

$(2x+3)(2x-3)=0$

$\boxed{x=\frac{-3}{2}}$ $\boxed{x=\frac{3}{2}}$

$2x^2+7x+3=0$

Adapted sum/product rule

Sum: 7 Product: $2\times3=6$

→ 6 and 1

Use this to split the middle term

$2x^2+6x+1x+3=0$

Factor by grouping

$(2x^2+6x)+(1x+3)=0$

$2x(x+3)+1(x+3)=0$

$(2x+1)(x+3)=0$

$\boxed{x=\frac{-1}{2}}$ $\boxed{x=-3}$

S | A
T | C

**Example**
Factor and solve the following TRIG equations.

$\tan^2\theta - 4\tan\theta + 3 = 0$ , $0 \le \theta < 2\pi$ ← radians

Imagine this is $x^2$ → Sum/Product rule
S: −4 P: +3

$(\tan\theta - 3)(\tan\theta - 1) = 0$

$\tan\theta = 3$ → Q1 or Q3
$\theta = \boxed{1.25}$ Q1
$\theta = 1.25 + \pi = \boxed{4.39}$ Q3

$\tan\theta = 1$ → Q1 or Q3
$\theta = \boxed{0.79}$ Q1
$\theta = 0.79 + \pi = \boxed{3.93}$ Q3

$2\sin^2\theta = 1$ , $0 \le \theta < 360^0$ ← degrees

$\sin^2\theta = \frac{1}{2}$

$\sin\theta = \pm\frac{1}{\sqrt{2}}$

$\sin\theta = \frac{1}{\sqrt{2}}$ Q1 or Q2
$\boxed{\theta = 45^\circ \text{ Q1}}$
$\boxed{\theta = 135^\circ \text{ Q2}}$

$\sin\theta = \frac{-1}{\sqrt{2}}$ Q3 or Q4
$\theta_R = 45^\circ$
$\boxed{\theta = 225^\circ \text{ Q3}}$ $\boxed{\theta = 315^\circ \text{ Q4}}$

$\csc^2\theta - 3\csc\theta - 10 = 0$ , $\theta \in R$ general solution needed. We can use radians or degrees, though!
S: −3 P: −10

$(\csc\theta - 5)(\csc\theta + 2) = 0$

$\csc\theta = 5$
$\sin\theta = \frac{1}{5}$ Q1 and Q2
$\theta = 11.5$ Q1
$\theta = 168.5$ Q2
→ $\boxed{11.5 + 360n}$
$\boxed{168.5 + 360n}$

$\csc\theta = -2$
$\sin\theta = \frac{-1}{2}$ → Q3 or Q4
$\theta_R = 30^\circ$
$\theta = 210^\circ$ Q3
$\theta = 330^\circ$ Q4
→ $\boxed{210 + 360n}$
$\boxed{330 + 360n}$

S | A
T | C

**Try**

radians

Q2 is excluded

1. Solve the following eqauations over the domain $-\pi \le \theta < \frac{\pi}{2}$ (rounded to 2 decimals)

**b.** $4\cos^2\theta - 3 = 0$

$\cos^2\theta = \frac{3}{4}$

$\cos\theta = \frac{\sqrt{3}}{2}$ Q1 and Q4

$\boxed{\theta = \frac{\pi}{3}}$

$\boxed{\theta = -\frac{\pi}{3}}$

b. $2\tan^2\theta = 3$

$\tan^2\theta = \frac{3}{2}$

$\tan\theta = \frac{\sqrt{3}}{\sqrt{2}}$ Q1 and Q3

not a special triangle

$\theta =$ $\boxed{0.89}$ rads

$\theta = -\pi + 0.89$

$\boxed{\theta = -2.25}$

**2.** Solve the following eqauations over the domain $-90^o \le \theta < 270^o$.

0 | 0
0 | 0

270-90

**c.** $2\cos\theta\sin\theta - \cos\theta = 0$

Divide by $\cos\theta$

Mind that

$2\cos\theta\sin\theta = \cos\theta$

$2\sin\theta = 1$

$\sin\theta = \frac{1}{2}$

Q1 or Q2

$\boxed{\theta = 30^\circ}$

$\boxed{\theta = 150^\circ}$

$\cos\theta = 0$

$\boxed{\theta = 90^\circ}$

$\boxed{\theta = -90^\circ}$

Can also use graphing calculator.

b. $3\tan\theta + \tan^2\theta = 2\tan\theta$

$\tan^2\theta + \tan\theta = 0$

S: 1 P: 0

$\rightarrow (\tan\theta + 1)(\tan\theta) = 0$

$\tan\theta = -1$ Q2 and Q4

$\theta_R = 45^\circ$

$\boxed{\theta = 135^\circ}$ $\boxed{\theta = -45^\circ}$

$\theta_R = 0^\circ$

$\boxed{\cancel{\theta = 0^\circ}}$

$\boxed{\theta = 180^\circ}$

**3.** Solve the following eqauations over the domain $-2\pi \le \theta \le 2\pi$, then determine the general solution

Adapted S/P rule.

**d.** $2\cos^2\theta - \cos\theta - 1 = 0$

S: -1 P: -2 → -2 and 1

$2\cos^2\theta - 2\cos\theta + \cos\theta - 1 = 0$

$2\cos\theta(\cos\theta - 1) + 1(\cos\theta - 1) = 0$

$(2\cos\theta + 1)(\cos\theta - 1) = 0$

$\cos\theta = -\frac{1}{2}$ Q2, Q3

$\theta_R = \frac{\pi}{3}$

$\boxed{\theta = \frac{2\pi}{3}, \frac{4\pi}{3}, -\frac{2\pi}{3}, -\frac{4\pi}{3}}$

$\cos\theta = 1$

$\boxed{\theta = 0, 2\pi, -2\pi}$

General solution:

$\boxed{\theta = \frac{2\pi}{3}n, \ n \in I}$

b. $5\sin^2\theta + 3\sin\theta = 2$

S: 3 P: -10 → 5 and -2

$5\sin^2\theta - 2\sin\theta + 5\sin\theta - 2 = 0$

$\sin\theta(5\sin\theta - 2) + 1(5\sin\theta - 2) = 0$

$(\sin\theta + 1)(5\sin\theta - 2) = 0$

$\sin\theta = -1$

$\boxed{\theta = \frac{3\pi}{2}, -\frac{\pi}{2}}$

$\sin\theta = \frac{2}{5}$ Q1, Q2

$\boxed{\theta = 0.4115, 2.7301, -3.5531, -5.8717}$

Messy general solution

Finally a normal domain

**4.** Solve the following eqauations over the domain $\boxed{0 \le \theta < 2\pi}$.

**e.** $4\tan^2\theta = 2 - 5\tan\theta$

$4\tan^2\theta + 5\tan\theta - 2 = 0$

S: 5 P: -8

Nothing can work. We must graph.

Set window for x to $[0, 2\pi]$

Graph this equation. Make sure you're in RAD.

Set window for y to something short like $[-5, 5]$.

2nd, TRACE, Zero.

Find x-intercepts.

$\boxed{\theta = 0.31, 2.14, 3.45, 5.28}$

b. $4\sin\theta + 3 = 2\sin^2\theta$

$2\sin^2\theta - 4\sin\theta - 3 = 0$

S: -4 P: -6

Again not possible.

Graph.

$\boxed{\theta = 3.76, 5.66}$

*Graphing can be used to solve any trig equation. One downside is that it doesn't give exact values for radians.

Works awesome for degrees, though!

**Answers**

1. a) $\theta = \pm\frac{\pi}{6}, -\frac{5\pi}{6}$ b) $\theta = \pm 0.89, -2.26$
2. a) $\theta = 30^o,\ \pm 90^o, 150^o$ b) $\theta = 0^o,\ 180^o,\ 135^o$
3. a) $\theta = 0,\ \pm\frac{2\pi}{3}, \pm\frac{4\pi}{3},\ \pm 2\pi$

   $\theta = \frac{2\pi}{3}n,\ n \in I$

   b) $\theta = -5.87,\ -3.55,\ -\frac{\pi}{2},\ 0.41,\ 2.73,\ \frac{3\pi}{2}$

   $\theta = 0.41 + 2\pi n,\ n \in I\ \ OR\ \ \theta = 2.73 + 2\pi n,\ n \in I\ \ OR\ \ \theta = \frac{3\pi}{2} + 2\pi n,\ n \in I$

4. a) $\theta = 0.31,\ 2.14,\ 3.45, 5.28$ b) $\theta = 3.76,\ 5.66$

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**Chapter 4 Skills Organizer**

| Process | | Example | Things to Remember |
|---|---|---|---|
| Converting Angle Measures | From Degrees to Radians<br><br>From Radians to Degrees | | |
| Determining Coterminal Angles | | | |
| Determining the Six Trig Ratios for Angles in the Unit Circle | | | |
| Solving Trig Equations | For a restricted domain<br><br>A general solution | | |

**Textbook Page 215-219 – Review and Practice Test**