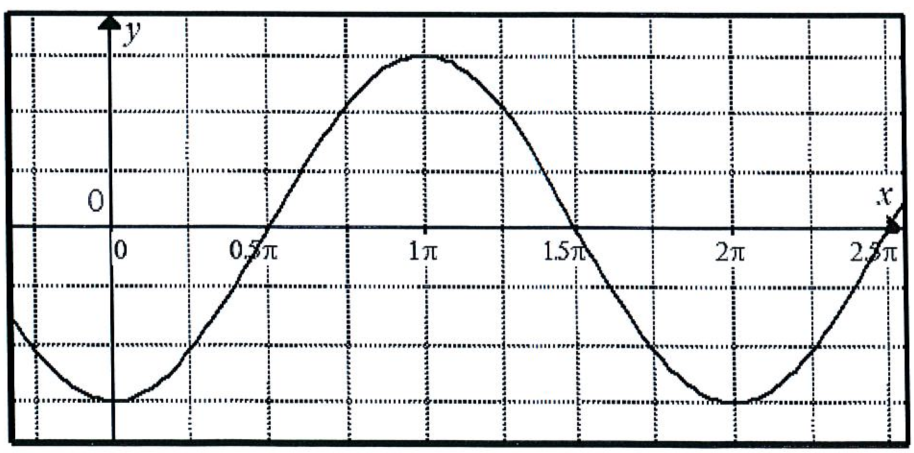
Key

MATH 30-1

CHAPTER 5
TRIG FUNCTIONS
&
GRAPHS



Math 30-1
Unit: **Trigonometry Functions and Graphs**
Topic: **Graphing Sine and Cosine Functions**

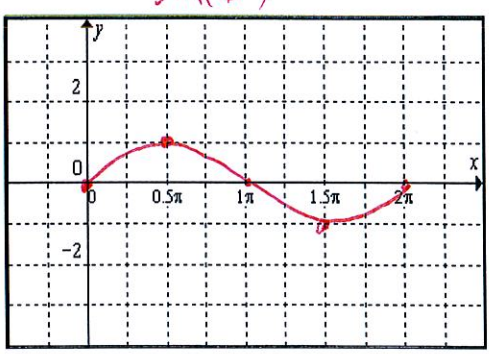
Objectives:

- *To sketch the graphs of y = sin x and y = cos x*
- *To determine the characteristics of the graphs of y = sin x and y = cos x*
- *To demonstrate an understanding of the effects of vertical and horizontal stretches on the graphs of sinusoidal functions*
- *To solve a problems by analyzing the graph of a trigonometric function*

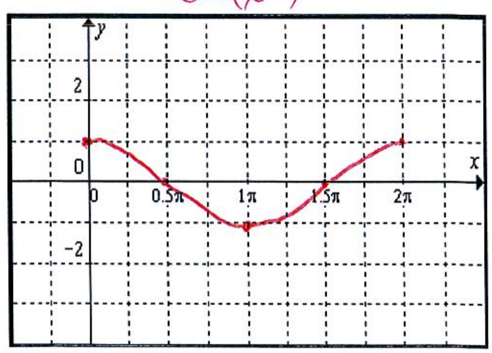
Periodic Functions:
A periodic function is a function whose graph repeats regularly over some interval of the domain.

Graph $y = \sin\theta$ and $y = \cos\theta$ for $0 \le \theta \le 2\pi$

sin(Ø)



cos(Ø)

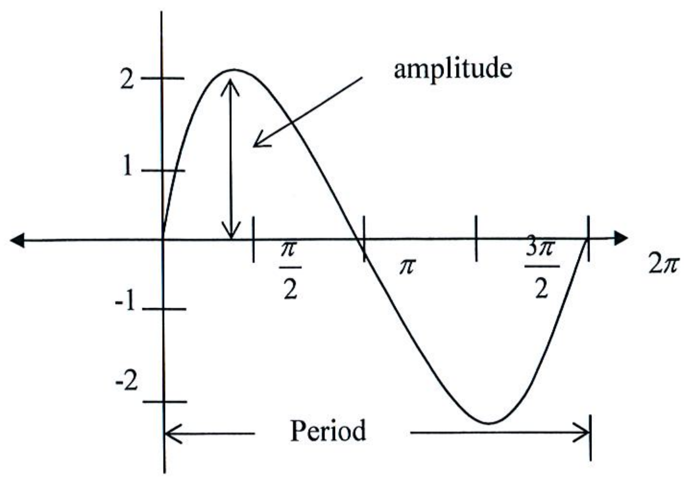


They're pretty much the same.

It's like cos(Ø) is sin(Ø) but shifted left by $\frac{\pi}{2}$

Or, sin starts at y=0, cos starts at y=1

Graph $y = 2\sin\theta$ for $0 \le \theta \le 2\pi$



For $y = a\sin\theta$

As a increases or decreases, the period stays the same and the amplitude increases or decreases.
a is a vertical stretch.

Example

Compare the graphs given $0 \le \theta \le 2\pi$

a. $y = \sin\theta$ and $y = 3\sin\theta$ Any changes in domain, period, range or amplitude?

Range changes from $-1 \le y \le 1$ to $-3 \le y \le 3$

Amplitude changes from 1 to 3.

b. $y = \cos\theta$ and $y = -\frac{1}{2}\cos\theta$ Any changes in domain, period, range or amplitude?

Range changes to $-\frac{1}{2} \le y \le \frac{1}{2}$

Amplitude changes to $\frac{1}{2}$

Note

- Vertical stretches of the sine and cosine graphs work exactly the same way as they did for the functions we graphed in our translations unit.
- Remember that vertical stretches affect only the y-coordinates, not the x coortinate.
- Vertical stretches affect amplitude and the range.

Your a-value is your amplitude.

Example

Compare the graphs given $0 \le \theta \le 2\pi$

a. $y = \sin\theta$ and $y = \sin(4\theta)$ Any changes in domain, period, range or amplitude?

Period changes from 2π to $\frac{\pi}{2}$

b. $y = \cos\theta$ and $y = \cos\left(\frac{1}{2}\theta\right)$ Any changes in domain, period, range or amplitude?

Period changes to 4π

Note

- Horizontal stretches also work the same way as before. Since these stretches affect the x coordinates, they change the period of the sine or cosine function.
- Horizontal stretches affect period
- The period of the sine and cosine functions will always be $\frac{2\pi}{b}$ or $\frac{360}{b}$

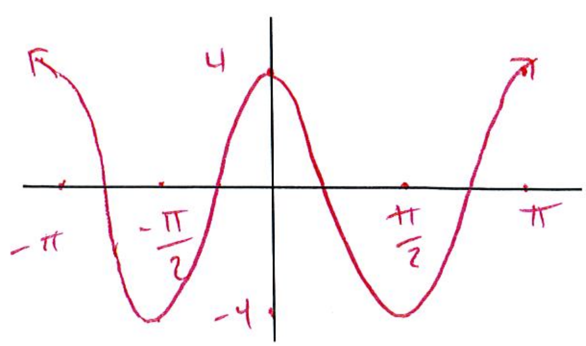
Example

Consider the graph of $y = 4\cos 2x$. $0 \le x \le 2\pi$

a. State the amplitude and period.

Amplitude = 4

Period = $\frac{2\pi}{2} = \pi$



b. Sketch the graph.

Example: Complete the Table ($0 \le \theta \le 2\pi$)

Equation	Describe transformations to graph of $y = \cos\theta$ or $y = \sin\theta$	Amplitude & Range	Period in degrees	Period in radians	x - intercepts 1st cycle	y - intercept
$y = 2\sin 4\theta$	Vert str by 2 Horiz str by $\frac{1}{4}$	2 $[-2, 2]$	$90°$	$\frac{\pi}{2}$	$0, \frac{\pi}{4}, \frac{\pi}{2}$	0
$y = 4\cos 3\theta$	Vert str by 4 Horiz str by $\frac{1}{3}$	4 $[-4, 4]$	$120°$	$\frac{2\pi}{3}$	$\frac{\pi}{6}, \frac{\pi}{2}$	4
$y = -\sin 6\theta$	Vert ref. Horiz str. by $\frac{1}{6}$	1 $[-1, 1]$	$60°$	$\frac{\pi}{3}$	$0, \frac{\pi}{6}, \frac{\pi}{3}$	0
$y = -5\sin\frac{1}{2}\theta$	Vert ref. Vert str 5 Horiz str 2	5 $[-5, 5]$	$720°$	4π	$0, 2\pi, 4\pi$	0
$y = 6\cos\left(-\frac{1}{3}\theta\right)$	Vert str 6 Horiz refl. Horiz str by 3	6 $[-6, 6]$	$1080°$	6π	$\frac{3\pi}{2}, \frac{9\pi}{2}$	$+6$

Complete the table comparing $y = \sin x$ and $y = \cos x$

	$y = \sin x$	$y = \cos x$
Maximum	$(\frac{\pi}{2}, 1)$	$(0, 1)$
Minimum	$(\frac{3\pi}{2}, -1)$	$(\pi, -1)$
Amplitude	1	1
Period	2π	2π
y-intercept	$(0, 0)$	$(0, 1)$
x-intercept(s)	$0, \pi, 2\pi$	$\frac{\pi}{2}, \frac{3\pi}{2}$
domain	$x \in \mathbb{R}$	$x \in \mathbb{R}$
range	$[-1, 1]$	$[-1, 1]$

Textbook Page 233 #4-11

Math 30-1
Unit: **Trigonometry Functions and Graphs**
Topic: **Transformations of Sinusodal Functions**

Objectives:

- *Graph and transform sinusoidal functions*
- *Identify the domain, range, phase shift, period, amplitude, and vertical displacement of sinusoidal functions*

Recall the following equation from transformations:

$$y = af\left[b(x-c)\right]+d$$

What the letters *a, b, c,* and *d* mean in trig . . .

a Vertical stretch or Amplitude

b Horizontal stretch by $\frac{1}{b}$
The period is $\frac{2\pi}{b}$ or $\frac{360}{b}$

c Horizontal shift, also called a "phase shift"

d Vertical shift. Also, d = midline

Example: Complete the Table

Equation	transformations to graph of $y=\sin\theta$	range	x - intercepts in degrees $0 \le x \le 360°$	y - intercept
$y=\sin(x-30°)$	Horiz. shift 30° right	$[-1, 1]$	30°, 210°	$\sin(-30) = -\frac{1}{2}$
$y=\sin x+2$	Vert. shift 2 up	$[1, 3]$	None	2
$y=\sin(x+60°)-1$	60° left 1 down	$[-2, 0]$	30°	$\sin(60)-1 = \frac{\sqrt{3}}{2}-1 = \frac{\sqrt{3}-2}{2}$
$y-45=\sin(x-45°)$ $y=\sin(x-45°)+45$	45° right 45 up	$[44, 46]$	None	$\sin(-45)+45 = -\frac{1}{\sqrt{2}}+45 \approx 44.29$

Summary of the Effect of the Parameters a, b, c and d:

For: $y=a\sin\left[b(x-c)\right]+d$

$y=a\cos\left[b(x-c)\right]+d$

amplitude $=|a|=\dfrac{\max-\min}{2}$

Period $=\dfrac{360°}{|b|}$ (for degrees)

Period $=\dfrac{2\pi}{|b|}$ (for radians)

Horizontal phase shift = c

- to right if c > 0
- to left if c < 0

Vertical displacement = d

- up if d > 0, down if d < 0
- $d=\dfrac{\max+\min}{2}$

Example

Write the equations of the following.

a. A cosine function having a horizontal phase shift of $75°$ right:

$\cos(x - 75°)$

b. A sine function having a horizontal phase shift of $\frac{3\pi}{5}$ radians left, and a vertical displacement 4 units up.

$\sin\left(x + \frac{3\pi}{5}\right) + 4$

$d = -2$

c. A sine function that has a vertical displacement of 2 units down, a horizontal phase shift of 30° to the left , a period of 120° and an amplitude of 1.

$+30°$ $\quad 120 = \frac{360}{b} \rightarrow b = 3$ $\quad a = 1$

$\sin[3(x + 30°)] - 2$

vertical

d. A cosine function that has reflected in the x-axis, has had a horizontal phase shift of $\frac{\pi}{4}$ radians to the right, a period of 3 and an amplitude of 4. $a = 4$

$-\frac{\pi}{4}$ $\quad 3 = \frac{2\pi}{b} \rightarrow b = \frac{2\pi}{3}$

$-4\cos\left[\frac{2\pi}{3}\left(x - \frac{\pi}{4}\right)\right]$

Example

Equation	Describe transformations	amplitude	Period (use same units as in equation)	range
$y=2\cos\left[\frac{1}{4}\left(x-\frac{\pi}{12}\right)\right]+3$	Horiz. shift Right $\frac{\pi}{12}$ Horiz. str. by 4 Vert str by 2 Up 3	2	$\frac{2\pi}{1/4}$ $=8\pi$	$[1, 5]$
$y=-\sin\left[3(x+\pi)\right]-4$	Vert ref. Horiz str by $\frac{1}{3}$ Left π Down 4	1	$\frac{2\pi}{3}$	$[-5, -3]$
$y=\frac{1}{2}\cos\left(2\theta-90^\circ\right)+1$ $y=\frac{1}{2}\cos(2(\theta-45))+1$	Vert str by $\frac{1}{2}$ Horiz str by $\frac{1}{2}$ Right 45° Up 1	$\frac{1}{2}$	$\frac{360}{2}$ $=180^\circ$	$[0.5, 1.5]$
$y=\sin\left(4\theta-\pi\right)+5$ $y=\sin(4(\theta-\frac{\pi}{4}))+5$	Horiz str by $\frac{1}{4}$ Right $\frac{\pi}{4}$ Up 5	1	$\frac{2\pi}{4}$ $=\frac{\pi}{2}$	$[4, 6]$

Textbook Page 250 #1-7,11, 13

Math 30-1
Unit: **Trigonometry Functions and Graphs**
Topic: **Transformations of Sinusodal Functions**

Objectives:

- *Develop equations of sinusoidal functions, expressed in radian and degree measure, from graphs and descriptions*
- *Recognize that more than one equation can be used to represent the graph of a sinusoidal function*

Example

Determine the following characteristics for the graphs shown below. Then, write the equation of the function as both a sine AND cosine function and there is a minimal horizontal phase shift. Assume a scale of 1 on the y-axis.

Note it starts at a minimum → Cosine and negative a

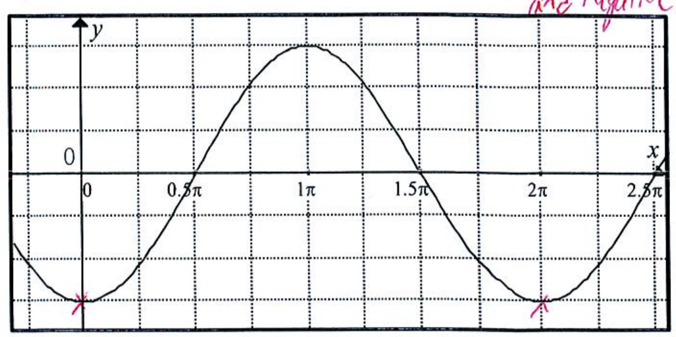
(a) i. Amplitude 3

a = +3

Period $= \frac{2\pi}{b}$

ii. Period 2π

b = 1



iii. Vertical displacement 0

d = 0

iv. Range $-3 \le y \le 3$

Sine function $a > 0$	Cosine function $a > 0$
Shifted $\frac{\pi}{2}$ right $y = 3\sin\left[1\left(x - \frac{\pi}{2}\right)\right]$	Shifted π right $y = 3\cos\left[1\left(x - \pi\right)\right]$

(b) i/ Amplitude 1

a = 1

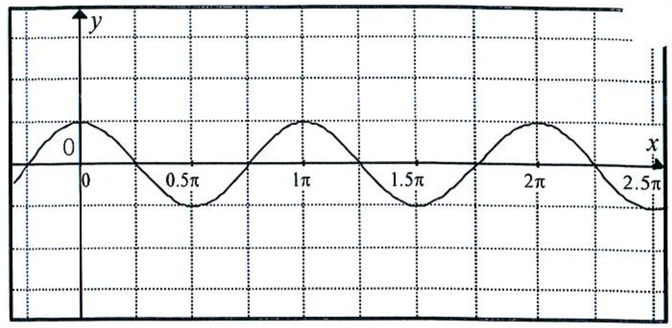
ii/ Period π

b = 2

iii/ Vertical displacement 0

d = 0

iv/ Range $-1 \le y \le 1$



Sine function $a > 0$	Cosine function $a > 0$
Shifted $\frac{\pi}{4}$ left $y = \sin\left[2\left(x + \frac{\pi}{4}\right)\right]$	$y = \cos[2x]$

Sine function $a < 0$	Cosine function $a < 0$
Shifted $\frac{\pi}{4}$ right $y = -\sin\left[2\left(x - \frac{\pi}{4}\right)\right]$	Shifted $\frac{\pi}{2}$ right $y = -\cos\left[2\left(x - \frac{\pi}{2}\right)\right]$

$\frac{\pi}{2} = \frac{2\pi}{b}$

$b = 4$

(c) i/ Amplitude 3

a = 3

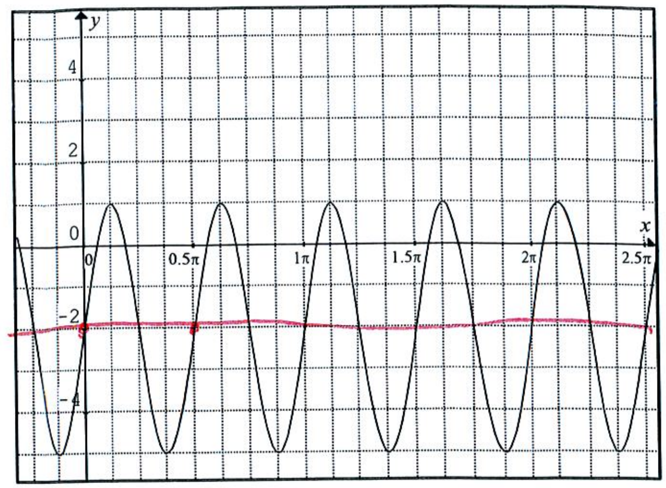
ii/ Period $\frac{\pi}{2}$

b = 4

iii/ Vertical displacement Down 2

d = -2

iv/ Range $-5 \leq y \leq 1$



Sine function $a > 0$	**Cosine function** $a > 0$
No shift left/right $y = 3\sin[4(x)] - 2$	Shifted right $\frac{\pi}{8}$ $y = 3\cos[4(x - \frac{\pi}{8})] - 2$
Sine function $a < 0$	**Cosine function** $a < 0$
Shifted right $\frac{\pi}{4}$ $y = -3\sin[4(x - \frac{\pi}{4})] - 2$	Shifted right $\frac{3\pi}{8}$ $y = -3\cos[4(x - \frac{3\pi}{8})] - 2$

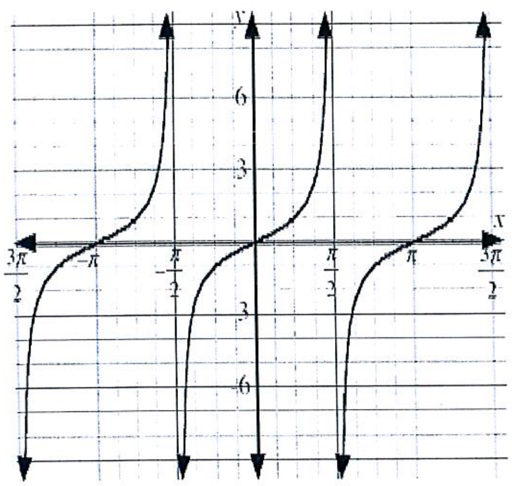
Textbook Page 252#14-16 and assignment

Math 30-1
Unit: **Trigonometry Functions and Graphs**
Topic: **The Tangent Function**

Objectives:

- *Learn to sketch the graph of* $y = \tan\theta$
- *Learn to determine the amplitude, domain, range and period of* $y = \tan\theta$
- *Learn to determine the asymptotes and x-intercepts for the graph of* $y = \tan\theta$
- *Learn to solve a problem by analyzing the graph of the tangent function*

Graph the function $y = \tan\theta$, for $-2\pi \le \theta \le 2\pi$. Describe the period, max/min values, the range, domain, vertical asymptotes x and y intercepts.



Window: $x:[-\frac{3\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{8}]$ $y:[-9,9,1]$

- Period is π
- there is no amplitude (or max/min)
- Range is $(-\infty, \infty)$
- Asymptotes at $\frac{\pi}{2} + \pi n, \ n \in I$
- Domain: $x \neq \frac{\pi}{2} + \pi n, \ n \in I$
- x-intercepts: $\pi n, \ n \in I$
- y-intercept at $y = 0$

The value of the tangent of an angle, θ , is the slope of the line passing through the origin and the point on the unit circle $(\cos\theta\ ,\ \sin\theta)$. You can think of it as a slope of the terminal arm of angle θ in standard position.

$$tan\theta = \frac{sin\theta}{cos\theta}$$

- When $\sin\theta = 0$, what is $\tan\theta$? For what values of θ is $\sin\theta = 0$?

$\tan\theta = 0$, $\theta = \pi n$, $n \in I$

- When $\cos\theta = 0$, what is $\tan\theta$? For what values of θ is $\cos\theta = 0$?

$\tan\theta =$ undefined, $\theta = \frac{\pi}{2} + \pi n$, $n \in I$

For tangent graphs, the distance between any two consecutive vertical asymptotes represents on complete period.

Example You might think $a=5$ means amplitude is 5, and that the period is $\frac{2\pi}{3}$

Graph $y = 5\tan 3\theta$ for $\left[-\frac{\pi}{2}, \frac{\pi}{2}, \frac{\pi}{4}\right][-3,3,1]$ and state the amplitude and period.

No amplitude. Period isn't super obvious (use x-ints), but it's $\frac{\pi}{3}$

Summary of the Effect of the Parameters a, b, c and d:

For $y = a\tan b\left[(x-c)\right] + d$

*

amplitude – not applicable
a value represents:

- a vertical stretch

Period $= \frac{180°}{|b|}$ (degree measure)

Period $= \frac{\pi}{|b|}$ (radian measure)

horizontal phase shift = c

- right if c > 0, left if c < 0

vertical displacement = d

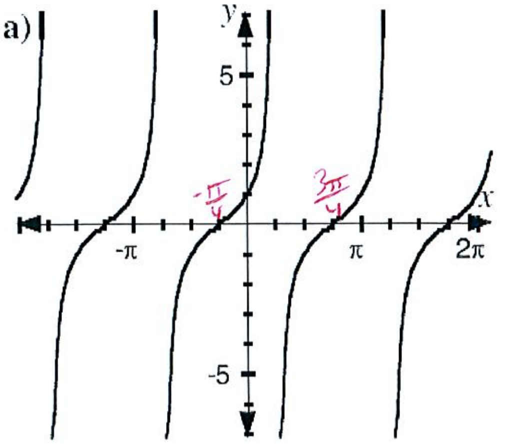
- up if d > 0
- down if d < 0

Note the main difference is the period.

The following graphs have an equation in the form $y = \tan b(x - c)$. Determine the equations of both graphs.

right shift

a)

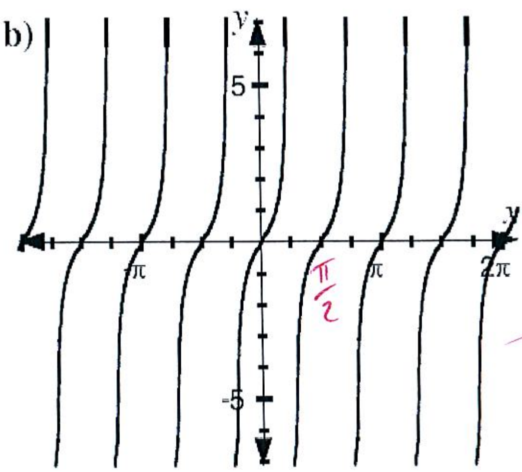


Period = π

$\rightarrow b = 1$

$\boxed{y = \tan\left(x - \frac{3\pi}{4}\right)}$

b)



No shift

Period = $\frac{\pi}{2}$

$\rightarrow \frac{\pi}{2} = \frac{\pi}{b} \rightarrow b = 2$

$\boxed{y = \tan 2x}$

Write the equation of a tangent function with a period of $\frac{\pi}{3}$ and a vertical displacement -3. d

$\frac{\pi}{3} = \frac{\pi}{b} \rightarrow b = 3 \rightarrow \boxed{y = \tan(3x) - 3}$

Textbook: pg. 262-265 #1-4, 6, 8, 9, 12

Math 30-1
Unit: **Trigonometry Functions and Graphs**
Topic: **Equations and Graphs of Trig Functions**

Objectives:

- *Learn to analyze a trigonometric function to solve a problem*
- *Learn to determine a trigonometric function that models a problem*
- *Learn to use a model of a trigonometric function for a real-world situation*

One of the most useful characteristics of trigonometric functions is their periodicity. With a partner, name as many situations in the world around us that can be represented in a sinusoidal graph.

Mathematics and scientists use the periodic nature of trigonometric functions to develop mathematical models to predict many natural phenomena.

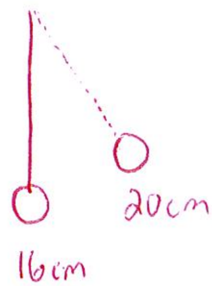
You can identify a trend or pattern, determine an approximate mathematical model to describe the process, and use it to make predictions (interpolate or extrapolate)

You can use graphs of trigonometric functions to solve trigonometric equations that model periodic phenomena, such as the swing of a pendulum, the motion of a piston in an engine, motion of a ferris wheel, variations in blood pressure, the hours of daylight throughout a year, and vibrations that create sounds.

Basically any repeating pattern.

Example

The pendulum of a grandfather clock swings with a periodic motion that can be represented by a trigonometric function. At rest, the pendulum is 16 cm above the base. The highest point of the swing is 20 cm above the base, and it takes 2 s for the pendulum to swing back and forth once. Assume that the pendulum is released from its highest point.



We can let y represent height and x represent time.

Midline = 18 cm = d → a = 2 cm

2s back and forth → period = 1 → b = 2π

a) Write a cosine equation that models the height of the pendulum as a function of time.

$$\boxed{y = 2\cos[2\pi(x)] + 18}$$

b) Write a sine equation that models the height of the pendulum as a function of time.

Only thing that changes is phase shift (c).

Move time "back" half a second.

→ "left" 0.5

$$\boxed{y = 2\sin[2\pi(x + 0.5)] + 18}$$

Example

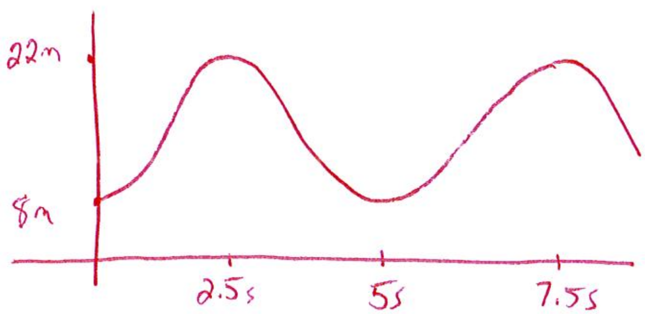
At the bottom of its rotation, the tip of the blade on a windmill is 8 m above the ground. At the top of its rotation, the blade tip is 22 m above the ground. The blade rotates once every 5 s. period

a) Draw one complete cycle of this scenario.

$\hookrightarrow 5 = \frac{2\pi}{b}$

$b = \frac{2\pi}{5}$

$d = 15 \quad a = 7$



b) A bug is perched on the tip of the blade when the tip is at its lowest point. Determine the cosine equation of the graph for the bug's height over time.

Cosine usually starts at top. We'll call this a phase shift. Right 2.5.

$$\boxed{y = 7\cos\left[\frac{2\pi}{5}(x - 2.5)\right] + 15}$$

c) What is the bug's height after 4 s?

$y = 12.8$ m

d) For how long is the bug more than 17 m above the ground?

Best way to do this is graph the equation from b, and also graph $y = 17$.
Find intersects, and figure how long it was above that height.

$x = 1.48 \qquad x = 3.52$

$\longrightarrow 3.52 - 1.48$

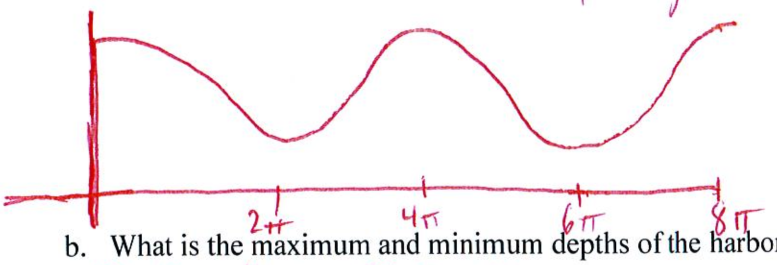
$= \boxed{2.04 \text{ seconds}}$

Example

The depth, d, in meters, of water in a harbor can be approximated by the equation $d(t) = 5\cos\left(\frac{1}{2}t\right) + 12$, where t is the time, in hours, after the first high tide.

a. Graph the function for two cycles. $b = \frac{1}{2} \to$ Period $= 4\pi$

Two cycles $\to 8\pi$



b. What is the maximum and minimum depths of the harbor?

12 is midline, 5 is amp.

$\to$ 17 max, 7 min

c. What is the period of the tide?

Period = 4π hours

Example

The motion of a point on an industrial flywheel can be described by the formula $h(t) = 13\cos\frac{2\pi}{0.7}t + 15$, where h is height, in metres, and t is the time, in seconds.

a. What is the range of the flywheel?

$2 \leq y \leq 28$

b. What is the period of the function?

$b = \frac{2\pi}{0.7} \to$ Period $= \frac{2\pi}{\left(\frac{2\pi}{0.7}\right)} \to = \boxed{0.7 \text{ hours}}$

c. What is the flywheel's height at 1 minute?

$t = 60$

$\boxed{y = 12.1 \text{ m}}$

Example

The fox population in a particular region can be modelled by the equation $F(t) = 500\sin\frac{\pi}{12}t + 1000$, where F is the fox population and t is the time, in months.

a. What is the maximum fox population throughout the year? What month(s) does the maximum number occur in? Max: 1500

Month 6

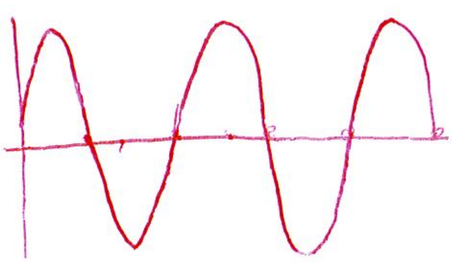
b. What is the period of the cycle?

$b = \frac{\pi}{12} \rightarrow \text{Period} = \frac{2\pi}{\pi/12} = \boxed{24 \text{ months}}$

Example

In some Caribbean countries, the current makes 50 complete cycles every second and the voltage is modeled by $V = 170\sin 100\pi t$.

a. Graph the voltage function over two cycles. Use the window settings given. Explain what
the scales on the axes represent.



x scale is time in seconds

y scale is voltage in volts

WINDOW
Xmin=■.001
Xmax=.05
Xscl=.01
Ymin=-200
Ymax=200
Yscl=20
Xres=1

b. What is the period of the current in these countries?

$\text{Period} = \frac{2\pi}{b} = \frac{2\pi}{100\pi} = \boxed{\frac{1}{50} \text{ seconds}}$

c. How many times does the voltage reach 110V in the first second?

Twice in one period.

50 periods in one second.

$\rightarrow \boxed{100 \text{ times}}$

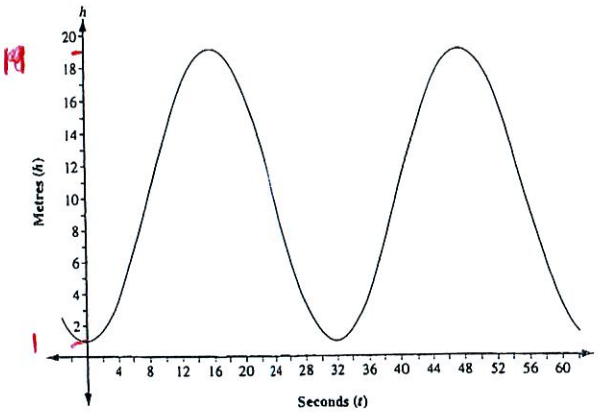
These are common on diploma exams

Example: The graph at right shows the height, h, in metres above the ground, over time, t, in seconds that it takes a ferris wheel to make one complete revolution of the ride. The maximum height of the ferris wheel is 19 m and the minimum height is 1 m.

Midline: $\frac{19+1}{2} = 10 = d$

Amplitude: $a = 9$

Period $= 32 = \frac{2\pi}{b} \rightarrow b = \frac{\pi}{16}$



(a) What is the period for 1 revolution of the ferris wheel?

$\boxed{\text{32 seconds}}$

(b) How high is the hub or centre of the ferris wheel off the ground?

Midline: $\boxed{10\text{ m}}$

(c) Write an equation for the height, h, of the ferris wheel as a function of time, t. Use the sine function for h in terms of t. Sine starts at midline. Note the period is 32 and we start at minimum. Divide 32 by 4 → 8 is our phase shift. right

$\rightarrow y = 9\sin\left[\frac{\pi}{16}(x-8)\right] + 10$

(d) find the distance from the ground, to the nearest tenth of a metre, of a particular point on the ride at $t = 10s$.

$\boxed{13.4\text{ m}}$

(e) using technology, find the first time, to the nearest tenth of a second, that a particular point on the ferris wheel is 6 m above the ground.

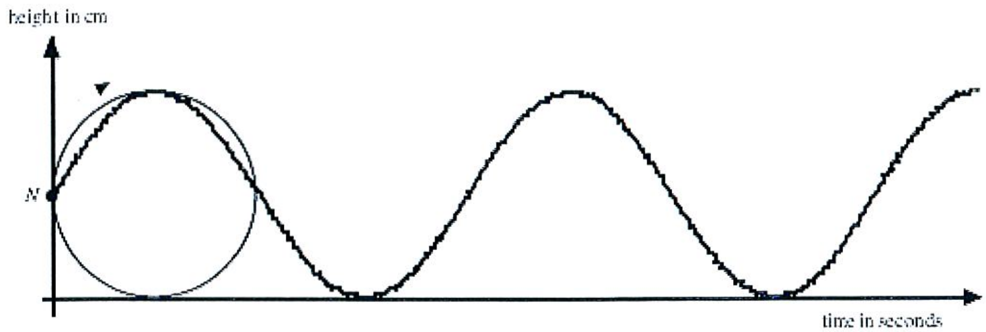
Set window like you see above.

Set $y_2 = 6$, solve for Intersect.

$\boxed{X = 5.7\text{ seconds}}$

Example

A nail is caught in the tread of a rotating tire at point N in the following sketch.



max height   Period = 6 seconds $= \frac{2\pi}{b}$ $b = \frac{\pi}{3}$

The tire has a diameter of 50 cm and rotates at 10 revolutions per minute. After 4.5 seconds the nail touches the ground.

Min height = 0   Midline 25 = d   Amplitude: $a = 25$

a. Determine an equation for the height of the nail as a function of time in the form $h(t) = a\sin bt + d, \quad a > 0$.

$h(t) = 25\sin\left(\frac{\pi}{3}t\right) + 25$

b. How far, to the nearest tenth of a centimeter, is the nail above the ground after 6.5 seconds?

$\boxed{h = 37.5 \text{ cm}}$

Textbook: pg. 275-279 #1-6, 9, 10, 12, 13, 15, 16, 19 and handout

Special triangles:

Math 30-1
Unit: **Trigonometry Functions and Graphs**
Topic: **Equations and Graphs of Trig Functions**

S	A
T	C

Objectives:

- *Learn to use the graphs of trigonometric functions to solve equations*

Example

Solve the following first degree trig equations.

a. $\sin\theta = \frac{1}{\sqrt{2}}, \frac{opp}{hyp}$ $0 \le x < 2\pi$ as exact values.

That's your hint that special triangles or Pythagoras are involved.

$\theta_R = 45^\circ = \frac{\pi}{4}$

CAST: Q1 or Q2

$\boxed{\frac{\pi}{4}}$ or $\boxed{\frac{3\pi}{4}}$

b. $\tan\theta = -5.24$, $-360^\circ \le x < 360^\circ$.

No "exact value" business. This is nice. Graph this, with the domain as your window. Find x-ints, Degree mode!

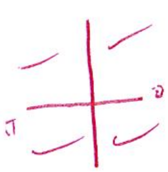
$\tan\theta + 5.24 = 0$

$\boxed{\theta = -259^\circ, -79^\circ, 101^\circ, 281^\circ}$

Alternatively, you could use inverse tan to get your reference angle. Mind CAST rule! Whatever you find easier!

c. $\sqrt{3}\sec\theta + 2 = 0$, $-\pi \le x < \pi$ rounded to the nearest hundredth.

Can't go in your calculator directly.



$\rightarrow \sqrt{3}\sec\theta = -2$

$\sec\theta = \frac{-2}{\sqrt{3}} \rightarrow \cos\theta = \frac{-\sqrt{3}}{2}$ ← special triangle!

$\theta_R = 30^\circ = \frac{\pi}{6}$

$\pi - \frac{\pi}{6} = \boxed{\frac{5\pi}{6}}$ $\quad -\pi + \frac{\pi}{6} = \boxed{\frac{-5\pi}{6}}$

You can solve these graphically or by hand. whatever you prefer. By hand is better for "exact value" questions.

Example

Solve the following second degree trig equations.

a. $4sin^2x - 3 = 0$, $0° \leq x \leq 360°$.

$\sin^2 x = \frac{3}{4} \rightarrow \sin x = \frac{\pm\sqrt{3}}{2} \leftarrow$ any quadrant

$x_R = 60°$

$\boxed{x = 60°, 120°, 240°, 300°}$

b. $2\cos^2 x + 3\cos x + 1 = 0$, $-\pi \leq x < 2\pi$, as exact values. By hand

yikes

Adapted S/P rule

$(2\cos^2 x + 2\cos x) + (1\cos x + 1) = 0$

$2\cos x(\cos x + 1) + 1(\cos x + 1) = 0$

$(2\cos x + 1)(\cos x + 1) = 0$

Overlap | Overlap

$\cos x = \frac{-1}{2}$

Special triangle

$\theta_R = 60° = \frac{\pi}{3}$

CAST: Q2 or Q3

Q2: $\boxed{\frac{2\pi}{3}}$

Q3: $\boxed{\frac{4\pi}{3}}$ and $\boxed{\frac{-2\pi}{3}}$

$\cos x = -1$

$x = 180° = \pi$

$\boxed{\pi}$ $\boxed{-\pi}$

c. $\tan^2 x + \tan x = 0$. Express your answer as a general solution in degrees.

$\tan x(\tan x + 1) = 0$

S | A
T | C

$\tan x = 0$

$x = 0°, 180°$

$\boxed{x = 180n,\ n \in I}$

$\tan x = -1$

If you aren't certain, use inverse trig on your calc to check!

$x_R = 45°$

$x = 135°, 315°$

$\boxed{x = 135 + 360n,\ n \in I}$

$\boxed{x = 315 + 360n,\ n \in I}$

d. $\sec^2 x = 1,\ 0 \le x < 2\pi$ Express your answer as a general solution in radians as exact values.

$\sec x = \pm 1$

$\cos x = \pm 1$

$x = 0, \pi$

General solution: $\boxed{x = \pi n,\ n \in I}$

Textbook page 211# 1, 2, 4, 5, 7