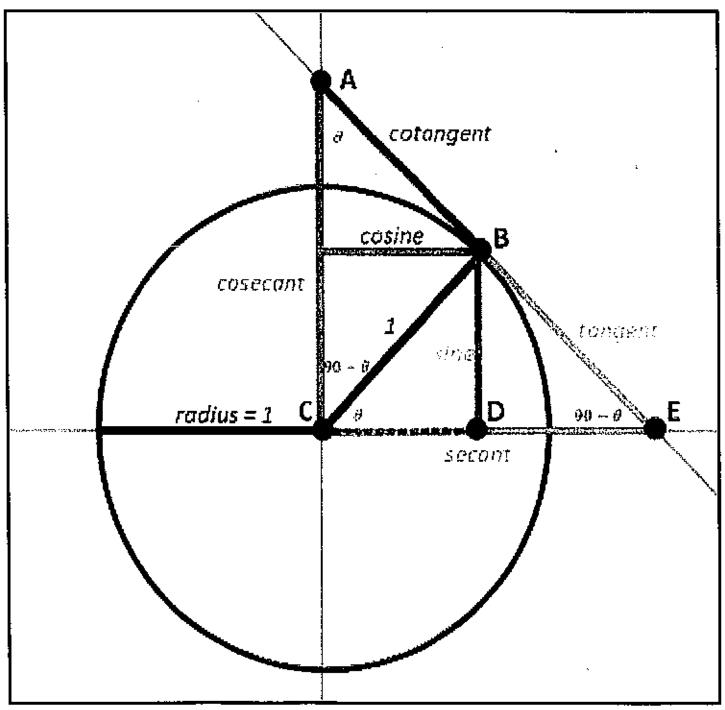
Key

MATH 30-1

CHAPTER 6 TRIGONOMETRIC IDENTITIES



$$\csc x = \frac{1}{\sin x} \qquad \sec x = \frac{1}{\cos x} \qquad \cot x = \frac{1}{\tan x}$$

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

$$\sin^2 x + \cos^2 x = 1 \qquad 1 + \tan^2 x = \sec^2 x \qquad 1 + \cot^2 x = \csc^2 x$$

$$\sin(A+B) = \sin A \cos B + \sin B \cos A$$
$$\sin(A-B) = \sin A \cos B - \sin B \cos A$$
$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$
$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\sin(2A) = 2\sin A \cos A$$
$$\cos(2A) = \cos^2 A - \sin^2 A$$

Math 30-1
Unit: **Verifying Trigonometric Identities**
Topic: **Reciprocal, quotient and Pythagorean identities**
Objectives:

- *Learn to verify a trigonometric identity numerically and graphically using technology*
- *Learn to explore reciprocal, quotient, and Pythagorean identities*
- *To determine non-permissible values of trigonometric identities*
- *To explain the difference between a trigonometric identity and a trigonometric equation*

What is the difference between an equation and an identity?

Equations are true for some values of x.
Identities are true for all values of x.

Example

$2x^2 + 3 = 11$ is an equation. It is only true for certain values of the variable x. Determine the solutions to this equation.

$2x^2 = 8$

$x^2 = 4$

$\boxed{x = \pm 2}$

Example

$(x+1)^2 = x^2 + 2x + 1$ is an identity. It is true for all values of the variable x. Prove it.

$(x+1)(x+1) = x^2 + 2x + 1$

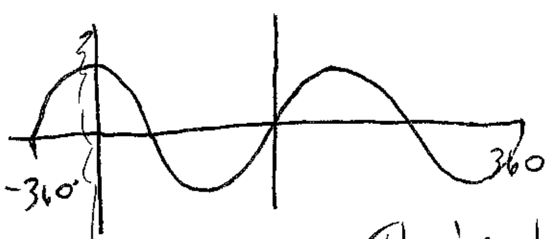
$x^2 + 2x + 1 = x^2 + 2x + 1$

Left hand side = right hand side
LHS = RHS

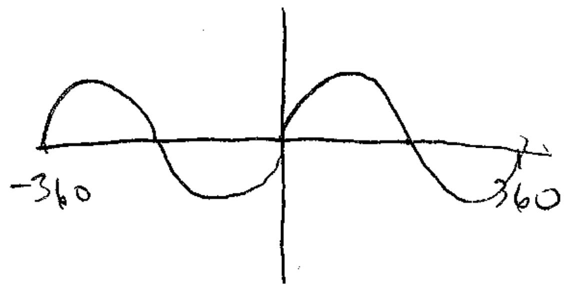
Comparing Two Trigonometric Expressions

1. Graph the curves $y = \sin x$ and $y = \cos x \tan x$ over the domain $\boxed{-360° \leq x < 360°.}$ Graph the curves on separate grids using the same range and scale. What do you notice?

sin x



cos x tan x



They're the same.

2. Use the table function on your calculator to compare the two graphs. Describe your findings.

2nd → WINDOW → Change ΔTbl to 90

→ Mostly the same, but cosx tanx has ERROR at ±90° and ±270°

3. Use your knowledge of *tan x* to simplify the expression *cos x tan x*

$$\cos x \tan x = \cos x \frac{\sin x}{\cos x}$$

$$= \boxed{\sin x}$$

NPVs: $\cos x \neq 0$

4. What are the non-permissible values of x in the equation *sin x = cos x tan x?*

$\cos x \neq 0$

$x \neq 90° + 180n, \quad n \in I$

The equations $y = \sin x$ and $y = \cos x \tan x$ are examples of trigonometric identities.

Trigonometric identities can be verified both numerically and graphically.

Numerically

Verify that $y = \sin x$ and $y = \cos x \tan x$ are equal for the following values of x.

a) $x = \pi$ Radians

$\sin \pi = 0$

$\cos \pi \tan \pi = 0$

b) $x = 60^\circ$ Degrees

$\sin 60 = 0.866 = \frac{\sqrt{3}}{2}$

$\cos 60 \tan 60 = 0.866 = \frac{\sqrt{3}}{2}$

c) $x = \frac{\pi}{4}$ Radians

$\sin\left(\frac{\pi}{4}\right) = 0.707 = \frac{\sqrt{2}}{2}$

$\cos\left(\frac{\pi}{4}\right) \tan\left(\frac{\pi}{4}\right) = 0.707 = \frac{\sqrt{2}}{2}$

Graphically

If you wanted to verify graphically that $y = \sin x$ and $y = \cos x \tan x$ are indeed the same thing, what would you expect to see on your calculator when you enter the two equations?

The two graphs would overlap perfectly $\rightarrow$ the same.

Example
For each of the following identities, state the restrictions and verify algebraically using the given values for θ.

a. $\tan\theta\cos\theta = \sin\theta$ for $\theta = \pi$.

$$\frac{\sin\theta}{\cos\theta} \cdot \cos\theta = \sin\theta$$

$\cos\theta \neq 0$

Verify: $0 = 0$

b. $\cos\theta(\sec\theta - 1) = 1 - \cos\theta$ for $\theta = \frac{\pi}{4}$.

$$\cos\theta\left(\frac{1}{\cos\theta} - 1\right) = 1 - \cos\theta$$

$\cos\theta \neq 0$

Verify: $0.293 = 0.293$

c. $\cot\theta\sec^2\theta - \cot\theta = \tan\theta$ for $\theta = \frac{\pi}{6}$.

$$\frac{1}{\tan\theta}\left(\frac{1}{\cos\theta}\right)^2 - \frac{1}{\tan\theta} = \tan\theta$$

$$\frac{\cos\theta}{\sin\theta}\left(\frac{1}{\cos\theta}\right)^2 - \frac{\cos\theta}{\sin\theta} = \frac{\sin\theta}{\cos\theta}$$

Verify: $0.577 = 0.577$

$\sin\theta \neq 0$, $\cos\theta \neq 0$

Textbook: pg. 296-297 #1-8, 10-14, 15

These are extra problems. Check with me if you want to check your answers.

Math 30-1
Verifying Trigonometric Identities

For each of the following identities:

1. State the restrictions on θ, and
2. Verify each identity for the value given.

1. $\cot\theta\sin\theta=\cos\theta$, $\theta=\frac{\pi}{3}$

2. $\frac{1}{\cos\theta}=\tan\theta\csc\theta$, $\theta=\frac{\pi}{4}$

3. $\sec\theta\sin\theta=\tan\theta$, $\theta=\pi$

4. $\cot\theta\sec^2\theta-\cot\theta=\tan\theta$, $\theta=\frac{\pi}{6}$

5. $\frac{1}{\sin\theta}=\sin\theta+\sin\theta\cot^2\theta$, $\theta=\frac{\pi}{2}$

6. $\sin\theta\sec^2\theta=\csc\theta\tan^2\theta$, $\theta=\frac{\pi}{3}$

7. $\sin\theta = \tan\theta\cos\theta$, $\theta = 2\pi$

8. $\cot\theta\sec\theta = \dfrac{1}{\sin\theta}$, $\theta = \dfrac{\pi}{6}$

9. $\dfrac{1}{\sin\theta} = \dfrac{\cot\theta}{\cos\theta}$, $\theta = \dfrac{\pi}{3}$

10. $\cos\theta\csc\theta = \cot\theta$, $\theta = \dfrac{\pi}{6}$

11. $\cos\theta = \cos\theta\csc^2\theta - \cos\theta\cot^2\theta$, $\theta = \dfrac{\pi}{4}$

12. $\tan\theta + \cot\theta = \dfrac{1}{\sin\theta\cos\theta}$, $\theta = \dfrac{\pi}{4}$

13. $\dfrac{1+\cos\theta}{\sin\theta} = \dfrac{\sin\theta}{1-\cos\theta}$, $\theta = \dfrac{3\pi}{2}$

14. $\csc\theta = \dfrac{\sec\theta + \csc\theta}{1+\tan\theta}$, $\theta = \dfrac{\pi}{4}$

Math 30-1
Unit: **Trigonometric Identities**
Topic: **Reciprocal, quotient and Pythagorean identities**
Objectives:

- *Learn to explore reciprocal, quotient, and Pythagorean identities*
- *To determine non-permissible values of trigonometric identities*
- *To explain the difference between a trigonometric identity and a trigonometric equation*

Reciprocal Identities:

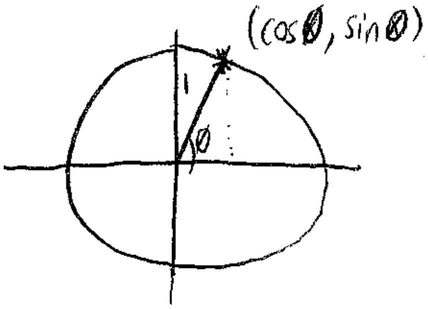
$\csc x = \dfrac{1}{\sin x}$ $\qquad \sec x = \dfrac{1}{\cos x}$ $\qquad \cot x = \dfrac{1}{\tan x}$

Quotient Identities:

$\tan x = \dfrac{\sin x}{\cos x}$ $\qquad \cot x = \dfrac{\cos x}{\sin x}$

Pythagorean Identity
Mark off a point in Quadrant I that is on the unit circle. Write the coordinates of that point in terms of sine and cosine. Apply the Pythagorean Theorem in the right triangle to establish the

Pythagorean Identity:



$\rightarrow \cos^2\phi + \sin^2\phi = 1^2$

$\rightarrow \cos^2\phi + \sin^2\phi = 1$

Pythagorean Identities: $\sin^2 x + \cos^2 x = 1 \quad 1 + \tan^2 x = \sec^2 x \qquad 1 + \cot^2 x = \csc^2 x$

These identities can be written in several ways:

For example: $\sin^2 x = 1 - \cos^2 x$

or $1 + \dfrac{\sin^2 x}{\cos^2 x} = \dfrac{1}{\cos^2 x}$ or $1 + \dfrac{\cos^2 x}{\sin^2 x} = \dfrac{1}{\sin^2 x}$

Verifying Identities for a Particular Case:

When verifying an identity we must treat the left side (LS) and the right side (RS) separately and work until both sides represent the same value.

This technique does not prove that an identity is true for all values of the variable – only for the value of the variable being verified.

Example

Verify that $\tan x = \dfrac{\sin x}{\cos x}$.

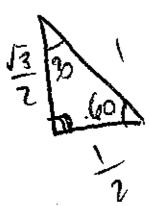
a. Determine the non-permissible values, in degrees, for the equation.

$\cos x \neq 0$

$\rightarrow x \neq 90^\circ + 180n, \; n \in I$

b. Verify using θ= 30°.

LHS: $\tan 30 = 0.577 = \dfrac{1}{\sqrt{3}}$



RHS: $\dfrac{\sin 30}{\cos 30} = \dfrac{1/2}{\sqrt{3}/2} = \dfrac{1}{\sqrt{3}} = 0.577.$

Change to an expression in terms of sin and cos. Sometimes other methods work better but at least it's a starting point.

Example
Express each as a single trig ratio.

a. $(\sec\theta)(\sin^2\theta)(\csc\theta)$

$= \frac{1}{\cos\theta}\cdot\sin^2\theta\cdot\frac{1}{\sin\theta}$

$= \frac{\sin^2\theta}{\sin\theta\cos\theta} = \frac{\sin\theta}{\cos\theta} = \boxed{\tan\theta}$

b. $\sin x+\cot x\cos x$

$\sin x + \frac{\cos x}{\sin x}\cdot\cos x$

$= \sin x + \frac{\cos^2 x}{\sin x}$

$= \frac{\sin^2 x+\cos^2 x}{\sin x} = \frac{1}{\sin x} = \boxed{\csc x}$

c. $\sec x\sin^2 x-\sec x$

$\frac{1}{\cos x}\cdot\sin^2 x - \frac{1}{\cos x}$

$= \frac{\sin^2 x - 1}{\cos x} = \frac{-\cos^2 x}{\cos x}$

$= \boxed{-\cos x}$

d. $\frac{\sin^2 x}{\cos^2 x}+1$

$\tan^2 x + 1$

$= \boxed{\sec^2 x}$

e. ~~$\sec x\sin^2 x-\sec x$~~

Duplicate

f. $\frac{1+\cot^2 x}{\sec^2 x}$

$= \frac{\csc^2 x}{\sec^2 x}$

$= \frac{\frac{1}{\sin^2 x}}{\frac{1}{\cos^2 x}} = \frac{\cos^2 x}{\sin^2 x}$

$= \boxed{\cot^2 x}$

Textbook: pg. 296-297 #1-8, 10-14, 15

Math 30-1
Unit: **Trigonometric Identities**
Topic: **Proving Identities**
Objectives:

- *To prove trigonometric identities algebraically*
- *To understand the difference between verifying and proving an identity*
- *To show that verifying the two sides of a potential identity are equal for a given value is insufficient to prove the identity*

To prove that an identity is true for all permissible values, it is necessary to express both sides of the identity in equivalent forms. One or both sides of the identity must be algebraically manipulated into an equivalent form to match the other side.
You cannot perform operations across the equal sign when proving a potential identity. Simplify the expressions on each side of the identity independently.

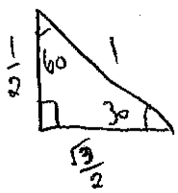
Hints in Proving an Identity:

1. Begin with the more complex side
2. If possible, use known identities given on the formula sheet, (ie. try to use the Pythagorean identities when squares of trigonometric functions are involved)
3. If necessary change all trigonometric ratios to sines and/or cosines, eg. replace *tan* x by $\frac{\sin x}{\cos x}$, or *sec* x by $\frac{1}{\cos x}$.
4. Look for factoring as a step in trying to prove an identity.
5. If there is a sum or difference of fractions, write as a single fraction
6. Occasionally, you may need to multiply the numerator or denominator of a fraction by its conjugate.

It is usually easier to make a complicated expression simpler than it is to make a simple expression more complicated.

Example

Consider the statement $\dfrac{1}{\cos x} - \cos x = \sin x \tan x$



a. **Verify** the statement is true for $x = \dfrac{\pi}{3}$ $\rightarrow 60^\circ$

L.S.

$\dfrac{1}{(1/2)} - \dfrac{1}{2}$

$= 2 - \dfrac{1}{2}$

$\boxed{= 1.5}$

R.S.

$\dfrac{\sqrt{3}}{2} \cdot \left(\dfrac{\frac{\sqrt{3}}{2}}{1/2} \right)$

$= \dfrac{\sqrt{3}}{2} \cdot \dfrac{2\sqrt{3}}{2}$

$= \dfrac{2 \cdot 3}{4} = \dfrac{6}{4} \boxed{= 1.5}$

b. **Prove** the statement algebraically:

L.S.

$= \dfrac{1}{\cos x} - \dfrac{\cos^2 x}{\cos x}$

$= \dfrac{1 - \cos^2 x}{\cos x}$

$\boxed{= \dfrac{\sin^2 x}{\cos x}}$

R.S.

$= \sin x \cdot \dfrac{\sin x}{\cos x}$

$\boxed{= \dfrac{\sin^2 x}{\cos x}}$

c. State the non-permissible values for x. Work in degrees.

$\cos x \neq 0$

$\therefore \boxed{x \neq 90 + 180n, \quad n \in I}$

Example

Prove the following identities. You should also be able to state restrictions for each identity.

a. $$\frac{2\tan x}{1+\tan^2 x} = \sin 2x$$

$$\frac{2\,\frac{\sin x}{\cos x}}{\sec^2 x} = \sin 2x$$

$$\to \frac{\frac{2\sin x}{\cos x}}{1/\cos^2 x} = \sin 2x$$

$$\to \frac{2\sin x\cos^2 x}{\cos x} = \sin 2x$$

$$2\sin x\cos x = \sin 2x$$

$$\boxed{2\sin x\cos x = 2\sin x\cos x}$$

← Identity

b. $$\frac{\sin\theta+\tan\theta}{1+\cos\theta} = \left(\frac{1}{\cot\theta}\right) \leftarrow \tan\theta$$

$$\frac{\sin\theta + \frac{\sin\theta}{\cos\theta}}{1+\cos\theta} = \frac{\sin\theta}{\cos\theta}$$

$$\to \frac{\frac{\sin\theta\cos\theta}{\cos\theta} + \frac{\sin\theta}{\cos\theta}}{1+\cos\theta} = \frac{\sin\theta}{\cos\theta}$$

$$\to \frac{\frac{\sin\theta(\cancel{\cos\theta+1})}{\cos\theta}}{(\cancel{1+\cos\theta})} = \frac{\sin\theta}{\cos\theta}$$

$$\boxed{\frac{\sin\theta}{\cos\theta} = \frac{\sin\theta}{\cos\theta}}$$

c. $$\frac{1}{1-\sin A} = \frac{1+\sin A}{\cos^2 A}$$ Conjugate?

$$\frac{(1+\sin A)}{(1-\sin A)(1+\sin A)} = \frac{1+\sin A}{\cos^2 A}$$

$$\to \frac{1+\sin A}{1-\sin^2 A} = \frac{1+\sin A}{\cos^2 A}$$

$$\to \boxed{\frac{1+\sin A}{\cos^2 A} = \frac{1+\sin A}{\cos^2 A}}$$

d. $$-\tan\theta = \frac{1-\tan\theta}{1-\cot\theta}$$

$$\frac{-\sin\theta}{\cos\theta} = \frac{1-\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}}$$

$$\to \frac{-\sin\theta}{\cos\theta} = \frac{\frac{\cos\theta-\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}}$$

$$\to \frac{-\sin\theta}{\cos\theta} = \frac{(\cos\theta-\sin\theta)(\sin\theta)}{(\sin\theta-\cos\theta)(\cos\theta)}$$

$$\to \boxed{\frac{-\sin\theta}{\cos\theta} = \frac{-\sin\theta}{\cos\theta}}$$

e. $\frac{\sin\theta+\cos\theta}{\csc\theta+\sec\theta}=\sin\theta\cos\theta$

$$\frac{\sin\theta+\cos\theta}{\frac{1}{\sin\theta}+\frac{1}{\cos\theta}} = \sin\theta\cos\theta$$

$$\rightarrow \frac{\cancel{\sin\theta+\cos\theta}}{\left(\frac{\cancel{\cos\theta+\sin\theta}}{\sin\theta\cos\theta}\right)} = \sin\theta\cos\theta$$

$$\rightarrow \boxed{\sin\theta\cos\theta = \sin\theta\cos\theta}$$

f. $\frac{\sec^2\theta}{\sec^2\theta-1}=\csc^2\theta$

$$\frac{1+\tan^2\theta}{\tan^2\theta} = \csc^2\theta$$

$$\rightarrow \frac{1}{\tan^2\theta}+1 = \csc^2\theta$$

$$\cot^2\theta+1 = \csc^2\theta$$

$$\boxed{\csc^2\theta = \csc^2\theta}$$

Example

Prove that $\frac{2\cos x-1}{2\cos^2 x-7\cos x+3}=\frac{1}{\cos x-3}$ is an identity.

Adapt. S/P rule

$\hookrightarrow (2\cos^2 x-\cos x)-(6\cos x+3)$

$\rightarrow \cos x(2\cos x-1)-3(2\cos x-1))$

$(\cos x-3)(2\cos x-1)$

$$\rightarrow \frac{\cancel{2\cos x-1}}{(\cos x-3)\cancel{(2\cos x-1)}} = \frac{1}{\cos x-3}$$

$$\rightarrow \boxed{\frac{1}{\cos x-3} = \frac{1}{\cos x-3}}$$

~~For what~~ values of x is this identity undefined?

$\cos x = 3$ and $\cos x = \frac{1}{2}$

N/A

$\boxed{x = 60^\circ, 300^\circ}$

S	A
T	C

Textbook: pg. 314-315 #1-8, 10-15, 16-18 & proofs worksheet on following page

Do these if you like. Check with me if you want to have them checked over.

Math 30-1
Trigonometric Proofs

On a separate piece of paper, algebraically prove each of the following identities by using any combination of the basic relations. You do not have to state the non-permissible values (even though you should).

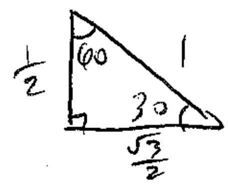
1. $\cot\theta\sin\theta = \cos\theta$
2. $\dfrac{1}{\cos\theta} = \tan\theta\csc\theta$
3. $\sec\theta\sin\theta = \tan\theta$
4. $\sin\theta = \tan\theta\cos\theta$
5. $\cot\theta\sec\theta = \dfrac{1}{\sin\theta}$
6. $\dfrac{1}{\sin\theta} = \dfrac{\cot\theta}{\cos\theta}$
7. $\cos\theta\csc\theta = \cot\theta$
8. $\dfrac{1}{\cos\theta} = \dfrac{\tan\theta}{\sin\theta}$
9. $\csc^2\theta\left(1-\cos^2\theta\right) = 1$
10. $1 = \cos^2\theta\left(1+\tan^2\theta\right)$
11. $\sin\theta\left(1+\cot^2\theta\right) = \csc\theta$
12. $\cos\theta = \sec\theta\left(1-\sin^2\theta\right)$
13. $\sin\theta\left(\csc\theta - \sin\theta\right) = \cos^2\theta$
14. $\sin^2\theta = \cos\theta\left(\sec\theta - \cos\theta\right)$
15. $\tan\theta\left(\cot\theta + \tan\theta\right) = \dfrac{1}{\cos^2\theta}$
16. $\cot^2\theta = \csc\theta\left(\csc\theta - \sin\theta\right)$
17. $\cot\theta\sec^2\theta - \cot\theta = \tan\theta$
18. $\dfrac{1}{\sin\theta} = \sin\theta + \sin\theta\cot^2\theta$
19. $\sin\theta\sec^2\theta = \csc\theta\tan^2\theta$
20. $\cos\theta = \cos\theta\csc^2\theta - \cos\theta\cot^2\theta$
21. $\tan\theta + \cot\theta = \dfrac{1}{\sin\theta\cos\theta}$
22. $\dfrac{1+\cos\theta}{\sin\theta} = \dfrac{\sin\theta}{1-\cos\theta}$
23. $\csc\theta = \dfrac{\sec\theta + \csc\theta}{1+\tan\theta}$
24. $\cos^3\theta\csc^3\theta\tan^3\theta = \csc^2\theta - \cot^2\theta$
25. $1 = \tan^2\theta\cos^2\theta + \cot^2\theta\sin^2\theta$
26. $\dfrac{\cos x}{1+\sin x} + \dfrac{\cos x}{1-\sin x} = 2\sec x$
27. $\sin^2 x = \dfrac{1-\cos 2x}{2}$
28. $1+\sin 2x = \left(\sin x + \cos x\right)^2$
29. $\sin 2x = 2\cot x\sin^2 x$
30. $\cos 2x = \dfrac{1-\tan^2 x}{1+\tan^2 x}$
31. $\sec^2 x = \dfrac{2}{1+\cos 2x}$
32. $\dfrac{1+\cos 2x}{\sin 2x} = \cot x$

Math 30-1
Unit: **Trigonometric Identities**
Topic: **Sum, difference and double angle identities.**
Objectives:

- *Learn to apply sum, difference, and double-angle identities to verify the equivalence of trigonometric expressions*
- *To verify a trigonometric identity numerically and graphically using technology*

Example

Use exact values to verify the following statement:



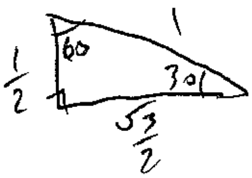
$$\sin(60+30)^\circ = \sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$$

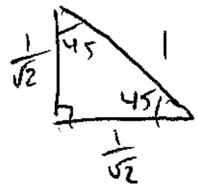
L.S. =	R.S. =
$\sin(90^\circ)$ $\boxed{= 1}$	$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{1}{2}$ $= \frac{3}{4} + \frac{1}{4}$ $\boxed{= 1}$

Sum and Difference Identities:

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$
$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$
$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$
$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$
$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$
$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

These are on the formula sheet





$\cos(\alpha - \beta)$ identity

Example: Write the expression $\cos 88° \cos 35° + \sin 88° \sin 35°$ as a single trigonometric function.

$= \cos(88 - 35)$

$= \boxed{\cos(53)}$

Example: Find the exact value of $\sin 15°$ 15° isn't in our special triangles. we can work with it, though.

$\sin 15 = \sin(45 - 30)$

$\longrightarrow \; = \sin 45 \cos 30 - \cos 45 \sin 30$

$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2} = \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} = \boxed{\frac{\sqrt{3}-1}{2\sqrt{2}}}$

Example: Determine the exact value of $\cos\frac{5\pi}{12}$ $\frac{\pi}{6} + \frac{\pi}{4}$

$= \cos\left(\frac{\pi}{6} + \frac{\pi}{4}\right)$

$= \cos\frac{\pi}{6}\cos\frac{\pi}{4} - \sin\frac{\pi}{6}\sin\frac{\pi}{4} = \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} - \frac{1}{2} \cdot \frac{1}{\sqrt{2}}$

Even I make mistakes

$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} = \boxed{\frac{\sqrt{3}-1}{2\sqrt{2}}}$

Example: Simplify each

a) $\sin 100° \cos 10° - \sin 10° \cos 100°$. ($\alpha$ β β α)

$= \sin(100 - 10)$

$= \sin 90$

$= 1$

b) $\cos\left(\frac{\pi}{2} - \theta\right)$

$= \cos\frac{\pi}{2}\cos\theta + \sin\frac{\pi}{2}\sin\theta$

$\boxed{= \sin\theta}$

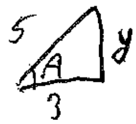
c) $\sin(\pi + x) - \sin(\pi - x)$

$(\sin\pi\cos x + \cos\pi\sin x) - (\sin\pi\cos x - \cos\pi\sin x)$

$= \cancel{\sin\pi\cos x} + \cos\pi\sin x - \cancel{\sin\pi\cos x} + \cos\pi\sin x$

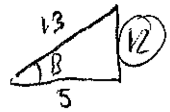
$= 2\cos\pi\sin x$

$= \boxed{-2\sin x}$



$5^2 - 3^2 = y^2$

$y = 4$



$\sin A = \frac{4}{5}$ $\qquad \sin B = \frac{-12}{13}$

Q1 $\qquad$ Q4

adj

Example: Given $\cos A = \frac{3}{5}$ and $\cos B = \frac{5}{13}$, where $0 \le A \le \frac{\pi}{2}$ and $\frac{3\pi}{2} \le B \le 2\pi$, find the exact value of $\cos(A+B)$.

hyp

$\cos A \cos B - \sin A \sin B$

$= \frac{3}{5} \cdot \frac{5}{13} - \frac{4}{5} \cdot \frac{-12}{13}$

$= \frac{15}{65} + \frac{48}{65} = \boxed{\frac{63}{65}}$

Double Angle Identities:

$$\sin 2A = 2\sin A\cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$$

These identities are on the formula sheet

Example

Express each in terms of a single trigonometric function:

$A = 4x$

a. $2\sin 4x\cos 4x$

$= \sin 2A$

$= \boxed{\sin 8x}$

b. $\cos^2 \frac{1}{2}A - \sin^2 \frac{1}{2}A$ "A" $= \frac{1}{2}A$

$\rightarrow \cos 2(\frac{1}{2}A)$

$= \boxed{\cos A}$

c. $\sin\frac{5}{2}x\cos\frac{5}{2}x$ Note that $\sin 2A = 2\sin A\cos A$

$A = \frac{5}{2}x$ therefore: $\frac{\sin 2A}{2} = \sin A\cos A$

$= \frac{\sin 2(\frac{5}{2}x)}{2}$

$= \boxed{\frac{\sin 5x}{2}}$

These are silly because they are given on your formula sheet.

Example

Determine an identity for $\cos 2A$ that contains only the sine ratio.

$$\cos 2A = \cos^2 A - \sin^2 A$$
$$\cos 2A = 1 - \sin^2 A - \sin^2 A$$
$$\boxed{\cos 2A = 1 - 2\sin^2 A}$$

Example

Determine an identity for $\cos 2A$ that contains only the cosine ratio.

$$\cos 2A = \cos^2 A - \sin^2 A$$
$$\cos 2A = \cos^2 A - (1 - \cos^2 A)$$
$$\cos 2A = \cos^2 A - 1 + \cos^2 A$$
$$\boxed{\cos 2A = 2\cos^2 A - 1}$$

Example

Simplify the expression $\dfrac{\sin 2x}{\cos 2x + 1}$ to one of the three primary trigonometric ratios

$$= \frac{2\sin x \cos x}{2\cos^2 x} = \frac{\sin x}{\cos x} = \boxed{\tan x}$$

Textbook: pg. 306-308 #1-11, 15-17, 20, 23

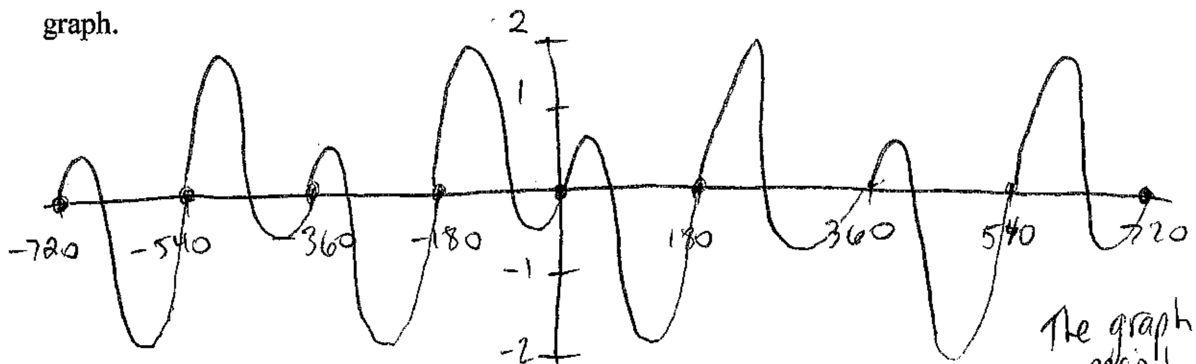
Math 30-1

Unit: **Trigonometric Identities**
Topic: **Solving Trig equations using Identities**
Objectives:

- *To solve trigonometric equations algebraically using know identities*
- *To determine exact solutions for trigonometric equations where possible*
- *To determine the general solution for trigonometric equations*
- *To identify and correct errors in a solution for a trigonometric equation*

Investigation

1. Graph the function $y = \sin 2x - \sin x$ over the domain $-720° < x \leq 720°$. Describe the graph.



The graph has a period of 360°

2. From the graph, determine an expression for the zeros of the function over the domain of all real numbers.

$\boxed{x = 180n, \quad n \in I}$ and $\boxed{x = 60 + 360n, \quad n \in I}$ and $\boxed{x = 300 + 360n, \quad n \in I}$

3. Algebraically solve $\sin 2x - sinx = 0$ over the domain. Compare your answers to the question above.

$= 2\sin x \cos x - \sin x = 0$

$\rightarrow \sin x(2\cos x - 1) = 0$

$\sin x = 0$ or $\cos x = \frac{1}{2}$ $\rightarrow$ $\boxed{x = \pm 60, \pm 300, \pm 420, \pm 660}$

$\downarrow$

$\boxed{x = \pm 0, \pm 360, \pm 720, \pm 180, \pm 540}$

To solve some trigonometric equations, you need to make substitutions using the trigonometric identities that you have studied in this chapter. This often involves ensuring that the equation is expressed in terms of one trigonometric function.

Example
Solve the following equation where $0 \le x \le 2\pi$.

$$2\cos^2 x - 3\sin x = 0$$

$2(1-\sin^2 x) - 3\sin x = 0$

$2 - 2\sin^2 x - 3\sin x = 0$

$0 = 2\sin^2 x + 3\sin x - 2 \quad \leftarrow S: 3 \quad P: -4$

$0 = (2\sin^2 x - \sin x) + (4\sin x - 2)$

$0 = \sin x(2\sin x - 1) + 2(2\sin x - 1)$

$0 = (\sin x + 2)(2\sin x - 1)$

$\sin x = -2$ N/A $\qquad \sin x = \frac{1}{2} \longrightarrow \boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}}$

S | A
T | C

Example

Solve for x as an exact value where $0 \le x \le 2\pi$

$$3 - 3\sin x - 2\cos^2 x = 0$$

$3 - 3\sin x - 2(1-\sin^2 x) = 0$

$3 - 3\sin x - 2 + 2\sin^2 x = 0$

$\rightarrow 2\sin^2 x - 3\sin x + 1 = 0 \quad \leftarrow S: -3 \quad P: 2$

$(2\sin^2 x - 2\sin x) - (\sin x + 1) = 0$

$2\sin x(\sin x - 1) - 1(\sin x - 1) = 0$

$(2\sin x - 1)(\sin x - 1) = 0$

$\sin x = 1$

$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}} \qquad \boxed{x = \frac{\pi}{2}}$

Example

Solve the equation $sin^2 x = \frac{1}{2}\tan x \cos x$ algebraically over the domain $0° \leq x < 360°$.

$$\sin^2 x = \frac{1}{2}\frac{\sin x}{\cos x}\cos x$$

$$\sin^2 x = \frac{1}{2}\sin x$$

$$2\sin^2 x = \sin x$$

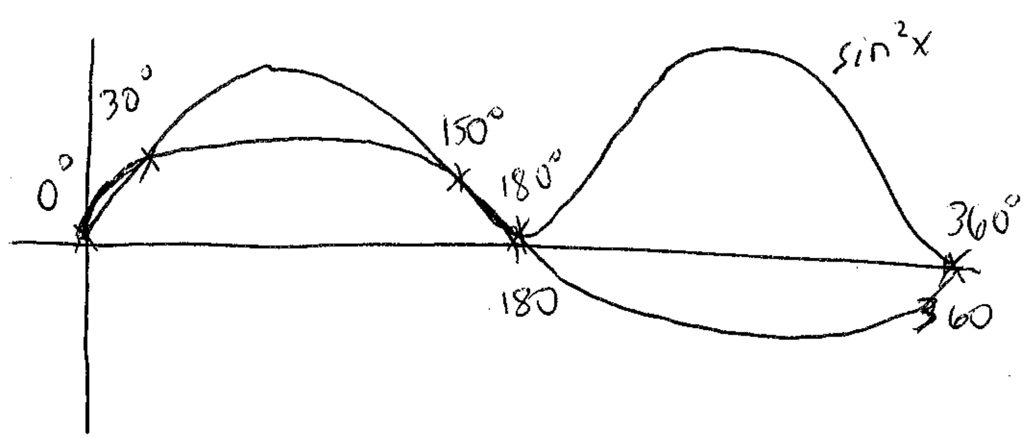
$$2\sin^2 x - \sin x = 0$$

$$\sin x(2\sin x - 1) = 0$$

$\sin x = 0$ $\quad$ $\sin x = \frac{1}{2}$

$\boxed{x = 0, 180°}$ $\quad$ $\boxed{x = 30°, 150°}$

Verify your answer graphically.



Example
Algebraically solve $\cos 2x = \cos x$. Give general solutions expressed in radians as exact values.

$2\cos^2 x - 1 = \cos x$

$2\cos^2 x - \cos x - 1 = 0$

$(2\cos^2 x - 2\cos x) + (\cos x - 1) = 0$

$2\cos x(\cos x - 1) + 1(\cos x - 1) = 0$

$(2\cos x + 1)(\cos x - 1) = 0$

S | A
T | C

$\cos x = -\frac{1}{2}$ $\qquad$ $\cos x = 1$

$\boxed{x = \frac{2\pi}{3}, \frac{4\pi}{3}}$ $\qquad$ $\boxed{x = 0}$

General: $x = 2\pi n$

$x = \frac{2\pi}{3} + 2\pi n$

$x = \frac{4\pi}{3} + 2\pi n$

$n \in I$

Example
Algebraically solve $3\cos x + 2 = 5\sec x$. Give general solutions expressed in radians as exact values.

$3\cos x + 2 = \frac{5}{\cos x}$

$3\cos x + 2 - \frac{5}{\cos x} = 0$ ← Multiply everything by cos x

$3\cos^2 x + 2\cos x - 5 = 0$ ← S: ~~NA~~ 2 P: −15

$(3\cos^2 x - 3\cos x) + (5\cos x - 5) = 0$

$3\cos x(\cos x - 1) + 5(\cos x - 1) = 0$

$(3\cos x + 5)(\cos x - 1) = 0$

$\cos x = -\frac{5}{3}$ ← N/A $\qquad$ $\cos x = 1$ → $x = 0$

General: $\boxed{x = 2\pi n, \ n \in I}$

Unless the domain is restricted, give general solutions. Check that the solutions for an equation do not include non-permissible values from the original equation.

Textbook: pg. 320-321 #1-11, 14-16, 17